

# Inequality, Aggregate Demand and the Crisis

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## Abstract

This paper discusses the impact of inequality on output and employment using a DSGE model. In particular, the model looks at the impact of lower bargaining power of workers on aggregate demand via the labour share of income. To address this issue, the model combines a search and matching model with Nash bargaining over income distribution with rule-of-thumb households, nominal price rigidities, CES production function and lower zero bound on monetary policy. The model features an endogenous labour share of income arising from the wage bargaining. Rule-of-thumb households create a transmission channel going from functional income distribution to consumption decisions, which feed back on aggregate demand through nominal price rigidities. Low substitution between labour and capital limits the increase in labour demand following a decline in wages, while the lower zero bound prevents an investment boom to overbalance the decline in consumption. The main result is that following a decline in the bargaining power of workers, the resulting drop in the labour share of income lowers consumption and aggregate demand. It follows that downward wage rigidities such as minimum wage limit the fall in output and employment. These results stand in contrast with standard New-Keynesian models.

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# 1 Introduction

This paper contributes to the literature connecting inequality and business cycle models. It presents a DSGE model in which rising inequality leads to an economic crisis through its negative effect on consumption and aggregate demand. Our contribution is to show that the transmission channel between inequality and macroeconomics takes place via aggregate demand effects rather than supply side or financial effects. This paper does not claim that these latter effects did not contribute to the actual crisis, but that the aggregate demand channel has been under-studied in the existing literature.

A number of contributions have documented the increase in inequality before the 2007 crisis. Piketty and Saez (2003) as well as Atkinson et al. (2011) have described the concentration of income in the top deciles of the population. Acemoglu (2003) and Lemieux (2006) review evidence and factors explaining increasing wage dispersion between high skilled and low skilled workers. Looking at functional income distribution, Blanchard (1997) and Blanchard and Giavazzi (2003) illustrate the decline in the labour share of income. In this paper, we are interested in the functional rather than the personal income distribution. Figure 1 shows that the labour share of income has declined in three fourth of 11 out of the 16 high income countries for which data are available over the period 1960-2010. Section 2 further details the decline in the labour share of income in high income countries as well as the factors behind this decline.

Two fundamentally different explanations for the fall in the labour share exists. The first claims that technological progress favored capital return, while the second identifies institutional changes as the cause of this development. All explanations require a departure from the assumption of unitary elasticity of substitution in the production function, which is usually made in RBC models. Choi and Rios-Rull (2009) and Arpaia et al. (2009) study the importance of the elasticity of substitution  $\zeta$  between factors of produc-

tion on their relative shares.<sup>1</sup> Institutional factors to explain a changing labour share can range from lower unionization rates to globalization pressure, but are usually modelled as a fall in worker's bargaining power.<sup>2</sup> This paper utilizes a general constant elasticity of substitution (CES) production function as well as a labour market featuring bargaining over wages and employment. As such, its model allows both for technological as well as institutional factors to affect the labour share of income. The focus of this paper lies in the impact of the institutional determinants on inequality and economic performance.

There are few attempts to link inequality and the crisis in the DSGE literature. In standard New Keynesian models with a search and matching friction and bargaining over income distribution, such as Sala et al. (2008), a decline in the labour share of income increases output and employment. The main reason is that lower wages increase labour demand by firms. Since the surplus from an additional match accruing to firms increases, they have an incentives to post more vacancies. A strong supply side effect follows, raising output. Additionally, changes in the income distribution have no effect on consumption and saving decisions since the representative household receives both labour and profit income.

The model presented in this paper differs from standard New Keynesian models along three dimensions. First, the model assumes a CES production function, which encompasses both Cobb-Douglas as well as Leontief production functions. Low degree of substitution between labour and capital implies that a decline in wages does not trigger a

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<sup>1</sup>Choi and Rios-Rull (2009) show that an elasticity of substitution below unity ( $\zeta = 0.75$  in their model) is required for a DSGE model to qualitatively reproduce empirical cyclical wage share properties in response to technology shocks. In contrast, Arpaia et al. (2009) take the presence of low- and high-skilled labour into account when investigating the impact of technical change and changes in the relative composition in the labour force.

<sup>2</sup>Berthold et al. (2002) model putty-clay technology and capital, thereby featuring low substitutability between capital and labour in the short run and high substitutability in the long run. Under these assumptions, a fall in workers' bargaining power will temporarily reduce the labour share. Blanchard and Giavazzi (2003) focus on firm entry costs and show that lower bargaining power, through raising employment, raises the number of firms and thereby competition in the market, thus lowering the price markup. By abstracting from aspects of technology and factor substitution, they establish that the labour share will ultimately be related to frictional costs, like entry costs, imposed on firms.

large increase in labour demand. This effect both produces fluctuations in the labour share of income as well as reduces the size of the transmission channel going from wages to labour demand.

Second, the model creates a channel between income distribution and consumption / saving decisions using households' heterogeneity. A first type of household is optimizing and makes consumption, saving and investment decisions to smooth inter-temporal consumption based on its permanent income. Additionally, a second type of household, called rule-of-thumb household, has no access to financial markets, and thus no saving or borrowing.<sup>3</sup> This household relies exclusively on labour income when employed or the replacement wage when unemployed. Contrary to standard New Keynesian models featuring exclusively optimizing representative households, this the presence of rule of thumb households allows a transmission channel going from income distribution to consumption decision and aggregate demand.

Third, the paper introduces the possibility of a liquidity trap implemented with a lower bound on the nominal interest rate, as in Christiano et al. (2009). In a liquidity trap, a shortage of demand, causing deflation, cannot be met by a fall in the nominal interest rate. As a result, the real interest rate rises, further lowering consumption and investment demand. This paper shows that the negative aggregate demand effect caused by lower workers' bargaining power far outweighs the positive supply effects in a liquidity trap. This channel is especially relevant in the 2008 crisis, when the collapse of the financial system required central banks to reduce interest rates to very low levels.

Summarizing, this paper highlights the importance of labour income as a driver of aggregate demand. Additionally, the relevance of this effect is underlined by conducting an experiment where a lower floor on real wages is introduced. Such a floor can be motivated by the downward rigidity of nominal wages, or by policy action in the form of

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<sup>3</sup>There is a large literature on rule of thumb households, see for instance Galí et al. (2007).

a minimum wage. Under such circumstances, the crisis induced by increased inequality is actually less severe since aggregate demand does not fall as strong.

The paper conceptually closest is Kumhof and Ranciere (2010). These authors also investigate the potential crisis-inducing impact of rising inequality. In their paper, workers react to falling wage income by increasing indebtedness, which eventually can cause an economic crisis. In contrast to this paper, Kumhof and Ranciere (2010) disregard aggregate demand effects by using a highly stylized model, assuming constant employment and abstracting from capital accumulation.

The next section reviews empirical evidence on the time series properties of the functional income distribution. Section 3 presents the mathematical derivation of the model used. Section 4 outlines the calibration strategy used, while Section 4.1 presents the simulation results. Finally, the last Section concludes.

## 2 Trends in the labour share of income

Figure 1 illustrates the decline in the labour share of income, which has taken place in the majority of 16 high income countries for which data exist over the period 1960-2010. The decline has been gradual and continuous in 11 out of 16 countries over the period 1960-2010. The largest drop took place in Ireland, Japan and Austria with an annual growth decline of -0.54 percent, -0.38 percent and -0.38 percent respectively. Italy, Norway and Finland displays declines around -0.30 percent annual, while the USA and Canada are close to -0.2 percent. More moderate declines took place in France -0.17 and Sweden -0.10. Interestingly, this decline has mainly taken place over the past two decades. In Australia and the Netherland, the labour share is constant but displays large increase in the 1970's and a long correction thereafter. Contrastingly, in Denmark and the UK, the labour share has been fluctuating around a constant trend. Lastly, Belgium is the sole country in which the labour share has been increasing at an annual rate of 0.17 percent.

[Figure 1 about here.]

These relatively large fluctuations in the labour share of income confirm existing studies. Blanchard (1997) and Blanchard and Giavazzi (2003) for instance make a similar analysis and point to labour market rigidities as a source of these fluctuations. However, this result is not consensual since Kaldor predicted that the labour share is constant around 65%. Picketty (2001) for instance makes a similar statement based on long term time series. The stability of the labour share of income is indeed a controversial issue. Solow (1958) argues that while constant at the aggregate level, the wage share displays excessive fluctuations at the sectoral level.<sup>4</sup>

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<sup>4</sup>See Young (2010) for a similar analysis on US data.

A reason to explain the absence of a consensus on the trends in the labour share of income is the measurement difficulties. A first issue is the treatment of quasi public administration and the financial sector in measuring the value added. A second issue is the contribution of stock options and the income of the self-employed in the compensation of labour. Askenazy (2003) shows that correcting for the self-employed as well as for quasi public administration affects the trends in the labour share significantly in France and the USA.

This section does not intend to engage in the debate of the long run properties of the labour share. However, since the under-shooting in the labour share is a short to medium run phenomena, it raises the question of the economic consequence of this deviation on aggregate demand and economic activity.

### 3 Model

#### 3.1 Households' heterogeneity and aggregate quantities

We define total number of households, consumption, employment and total labour endowment (labour supply) of all optimizing consumers as  $Y_{o,t}$ ,  $C_{o,t}$ ,  $N_{o,t}$  and  $L_{o,t}$ , respectively. For rule of thumb consumers, the appropriate quantities are  $Y_{r,t}$ ,  $C_{r,t}$ ,  $N_{r,t}$  and  $L_{r,t}$ . The total aggregate quantities are then given by the sums of these, thus  $C_t = C_{o,t} + C_{r,t}$  and the equivalents.

Consumption per household  $c_t$  is then given by

$$c_t = \frac{C_t}{Y_t} = \phi_c c_{o,t} + (1 - \phi_c) c_{r,t}, \quad (1)$$

where  $\phi_c = \frac{Y_{o,t}}{Y_t}$  is the share of optimizing consumers in the total population, and  $c_{i,t} = \frac{C_{i,t}}{Y_{i,t}}$   $i = [o, r]$ . We assume that each household has a maximum labour endowment of unity. We make the assumption that rule of thumb households fully use their labour endowment, thus  $L_{r,t} = Y_{r,t}$ , since it is their only source of income. For optimizing consumers, the capitalists, we assume that their labour supply can be a fraction  $v$  of unity, thus  $L_{o,t} = vY_{o,t}$ .<sup>5</sup>

The employment rate  $n$ , defined as employment over total labour endowment, is given by

$$\begin{aligned} n_t &= \frac{N_t}{L_t} = \frac{L_{o,t}}{L_t} n_{o,t} + \frac{L_{r,t}}{L_t} n_{r,t} \\ \Leftrightarrow n_t &= \phi_n n_{o,t} + (1 - \phi_n) n_{r,t}, \end{aligned} \quad (2)$$

where  $\phi_n = \frac{v\phi_c}{\phi_p}$  is the share of optimizing consumers in the workforce and  $\phi_p = 1 - (1 - v)\phi_c$ . When  $v = 1$ , then  $\phi_n = \phi_c$  and  $\phi_p = 1$ . The aggregate employment per household

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<sup>5</sup>We set this fraction exogenously. A model extension could have this value be determined endogenously, for example as a function of wealth.



is given by

$$\frac{N_t}{\bar{Y}_t} = \phi_p n_t, \quad (3)$$

Since only optimizing households hold capital, the capital stock per household  $k = \frac{K}{Y}$  is given by

$$k_t = \phi_c k_{o,t} \quad (4)$$

Furthermore, only optimizing households invest in capital, so that investment  $x$  per household is given by

$$x_t = \phi_c x_{o,t} \quad (5)$$

Equations 1 and 2 enable us to differentiate across 3 cases depending on the values of the parameters  $\phi_c$  and  $\nu$ . The first case is defined by  $\phi_c = 1$  and  $\nu = 1$  and corresponds to the standard New Keynesian model, with a single optimizing household and no rule of thumb household. The second case is defined by  $\phi_c < 1$  and  $\nu = 0$  and includes rule of thumb households. However, optimizing households do not participate in the labour market. There is a clear distinction between rule of thumb households receiving labour incomes and optimizing households receiving profit incomes. The third case is defined by  $\phi_c < 1$  and  $0 < \nu < 1$ . This is the middle ground case to the extent that both rule of thumb and optimizing households participate in the labour market and receive labour income.

## 3.2 Labour Market Flows

All workers not working in a period are unemployed and looking for a job. The pool of unemployed is given by  $u_t = 1 - n_{t-1}$ . Unemployed workers can be matched to a job and start working immediately in that period. The matching function is

$$m_t = \gamma_m u_t^\gamma v_t^{1-\gamma} \quad (6)$$

where  $m_t$  are new matches,  $v_t$  are posted vacancies,  $\gamma$  is the elasticity of matching to unemployed workers and  $\gamma_m$  is the overall matching efficiency. Workers are separated exogenously from their jobs with probability  $1 - \rho$ . Employment at  $t$  is given by the remaining stock of workers plus new matches.

$$n_t = \rho n_{t-1} + m_t \quad (7)$$

Thus, workers that had a job in period  $t - 1$  and loose it are immediately in the pool of unemployed and are able to find a job in period  $t$  again.

Three definitions are use to describe the labour market: the probability of filling a vacancy,  $q_t = m_t/v_t$ , the job finding probability  $p_t = m_t/u_t$  and the labour market tightness  $\theta_t = \frac{v_t}{u_t}$ . Furthermore, since the model assumes quadratic employment adjustment costs further below, we define the hiring rate  $h_t$  as

$$h_t = \frac{m_t}{n_{t-1}} = \frac{q_t v_t}{n_{t-1}} \quad (8)$$

Equation 7 can be expressed as a function of the hiring rate:

$$n_t = \rho n_{t-1} + h_t n_{t-1} \quad (9)$$

### 3.3 Households

Optimizing and rule of thumb households maximize their intertemporal utility function

$$\max U_{i,t} = \sum_{j=0}^{\infty} \beta^{t+j} u(c_{i,t+j}) \quad \text{for } i = [o, r], \quad (10)$$

where  $\beta$  is the time discount factor and the period utility function  $u(c_{i,t})$  is defined as

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\sigma^i}}{1-\sigma^i} \quad \text{for } i = [o, r].$$

Both types of households face the employment dynamics constraint.

$$n_{i,t} = \rho n_{i,t-1} + p_t(1 - n_{i,t-1}) \quad \text{for } i = [o, r]. \quad (11)$$

### 3.3.1 Rule of Thumb Households

Rule of thumb households do not have access to financial markets. Therefore, their budget constraint is given by their labour income plus their unemployment benefit payments  $w_u$ .

$$c_{r,t} = w_t n_{r,t} + w_u(1 - n_{r,t}) \quad (12)$$

The household maximizes its utility  $U_{r,t}$ :

$$U_{r,t} = \frac{c_{r,t}^{1-\sigma^r}}{1-\sigma^r} + \beta E_t \{U_{r,t+1}\} \quad (13)$$

subject to the employment and budget constraints, (11) and (12). The consumption of rule of thumb households is given by eq (12) since their budget constraint is binding. Furthermore, the marginal utility of consumption is given by

$$\lambda_{r,t} = c_{r,t}^{-\sigma^r} \quad (14)$$

The first derivative of  $U_{r,t}$  with respect to  $n_{r,t}$  yields

$$V_{r,t} = \lambda_{r,t}(w_t - w_u) + \beta E_t [V_{r,t+1}(\rho - p_{t+1})], \quad (15)$$

where  $V_{r,t}$  is the Lagrange multiplier on the employment dynamics constraint (11), and can thus be interpreted as the marginal utility value of a job to a household. It is useful to define the value of a job in terms of a consumption good, thus we define  $H_{n^r,t} = \frac{V_{r,t}}{\lambda_{r,t}}$ . We then obtain

$$H_{n^r,t} = w_t - w_u + \beta E_t [\Lambda_{t,t+1}^r (\rho - p_{t+1}) H_{n^r,t+1}], \quad (16)$$

where  $\Lambda_{t,t+1}^r = \frac{\lambda_{r,t+1}}{\lambda_{r,t}}$  is the stochastic discount factor for rule of thumb consumers.

### 3.3.2 Optimizing Consumers

Like rule of thumb consumers, optimizing consumers also earn labour income and unemployment benefits. These quantities have to be scaled by the relative labour market

participation  $v$  when expressing in per-household terms. Additionally, they can invest in bonds  $b_{o,t}$  paying a gross nominal interest rate  $R_{n,t}$ , and they can accumulate physical capital  $k_{o,p,t}$  subject to the accumulation function

$$k_{o,p,t} = (1 - \delta)k_{o,p,t-1} + x_{o,t}(1 - \phi_t), \quad (17)$$

where  $\phi_t$  are capital adjustment costs. The link between actual capital and physical capital depends on capacity utilization:

$$k_{o,t} = u_{k,t}k_{o,p,t-1} \quad (18)$$

Optimizing households are allowed to vary the usage of physical capital by the factor  $u_{k,t}$ , to earn a return  $u_{k,t}r_{k,t}$  on their physical capital stock. There is a cost  $\mathfrak{S}(u_{k,t})$  associated with capacity utilization.

The budget constraint of optimizing households is given by

$$\begin{aligned} c_{o,t} + x_{o,t} + \frac{b_{o,t}}{P_t} + \mathfrak{S}(u_{k,t})k_{o,p,t-1} \\ \leq w_t v n_{o,t} + w_u v (1 - n_{o,t}) + r_{k,t} u_{k,t} k_{o,p,t-1} + \frac{R_{n,t-1}}{P_t} b_{o,t-1} - \tau_{o,t} + \Pi_t, \end{aligned} \quad (19)$$

where  $\Pi_t$  are profit receipts from firms and  $P_t$  is the aggregate price level used to transform nominal bond values and returns to real values.

The household maximizes its utility  $U_{o,t}$ :

$$U_{o,t} = \frac{c_{o,t}^{1-\sigma^o}}{1-\sigma^o} + \beta E_t \{U_{o,t+1}\} \quad (20)$$

subject to the employment dynamics constraint (11), the budget constraint (19) and the capital accumulation (17), given the definitions of capital adjustment costs (52) and capital utilization costs (53). We define the Lagrange multipliers on the employment constraint as  $V_o$  (thus the marginal value of a job), the budget constraint as  $\lambda_o$  (thus the marginal utility of consumption), and the capital accumulation constraint as  $\varphi$  (thus the marginal value of a unit of capital, or Tobin's  $q$ ).

The first order conditions are given by

$$\lambda_{o,t} = c_{o,t}^{-\sigma^o} \quad (21)$$

$$\Lambda_{t,t+1}^o = \frac{1}{\beta} \frac{\pi_{t+1}}{R_{n,t}} \quad (22)$$

$$\varphi_t = \beta E_t (\Lambda_{t,t+1}^o [r_{k,t+1} u_{k,t+1} - \mathfrak{S}(u_{k,t+1}) + \varphi_{t+1} (1 - \delta)]) \quad (23)$$

$$\varphi_t \left[ 1 - \left( \phi_t + \frac{x_t^o}{x_{t-1}^o} \phi_t' \right) \right] = 1 - \beta E_t \left\{ \varphi_{t+1} \Lambda_{t,t+1}^o \left( \frac{x_{t+1}^o}{x_t^o} \right)^2 \phi_{t+1}' \right\} \quad (24)$$

$$r_{k,t} = \frac{\partial \mathfrak{S}(u_{k,t})}{\partial u_{k,t}} \quad (25)$$

$$V_{o,t} = \lambda_{o,t} v(w_t - w_u) + \beta E_t [V_{o,t+1} (\rho - p_{t+1})], \quad (26)$$

where  $\phi_t'$  is the first derivative of the capital adjustment cost function with respect to  $\left( \frac{x_t}{x_{t-1}} - 1 \right)$  and  $\Lambda_{t,t+1}^o = \frac{\lambda_{o,t+1}}{\lambda_{o,t}}$  is the stochastic discount factor for rule of thumb consumers and  $\pi_{t+1} = \frac{P_{t+1}}{P_t}$  is the gross rate of inflation.

Similarly to rule of thumb consumers, we define the value of a job in terms of a consumption good  $H_{n^o,t} = \frac{V_{o,t}}{\lambda_{o,t}}$ . We then obtain

$$H_{n^o,t} = v w_t - v w_u + \beta E_t [\Lambda_{t,t+1}^o (\rho - p_{t+1}) H_{n^o,t+1}], \quad (27)$$

### 3.4 The Wholesale Good Firm

Wholesale good firms produce output using capital and labour using a production function of the form  $Y_t^w = F(K_t, N_t)$ . Since we assume the production function to be homogeneous of degree one (as holds for CES functions), we can express output per household  $y_t^w = F(k_t, \phi_p n_t)$ . Firms sell their output to retail firms, who in turn sell goods to consumers at a markup. We define the relative price of wholesale products to final goods as  $p^w$ . The firm rents capital from the owners of the capital goods, hires labour and has to pay quadratic labour adjustment costs, which we define as  $(\kappa/2)h_t^2 n_{t-1}$ .

The firm maximizes its value  $F_t$ , expressed as per household, by choosing capital  $k$

and hiring  $h$ , subject to the dynamics governing employment (9). The value is given by

$$F_t = p_t^w y_t^w - w_t \phi_p n_t - \frac{\kappa}{2} h_t^2 n_{t-1} - r_t^k k_t + \beta E_t [\Lambda_{t,t+1}^o F_{t+1}], \quad (28)$$

where  $\Lambda_{t,t+1}^o$  is the firm's discount factor. The first order conditions with respect to  $k$ ,  $h$  and  $n$  (where we do not evaluate  $\partial h / \partial n$ ) are given, in that order, by

$$r_k = p_t^w a_t^k \quad (29)$$

$$\kappa h_t = J_t \quad (30)$$

$$J_t = p_t^w a_t^n - \phi_p w_t + \beta E_t \left[ \Lambda_{t,t+1}^o \frac{\kappa}{2} h_{t+1}^2 \right] + \beta \rho E_t [\Lambda_{t,t+1}^o J_{t+1}] \quad (31)$$

The marginal productivity of capital and labour are given by  $a^k$  and  $a^n$  respectively.  $J_t$  is the Lagrange multiplier on the "budget" constraint of employment dynamics (9), and thus can be interpreted as the marginal value to the firm of having another worker.

### 3.5 Bargaining

Firms and workers engage in Nash Bargaining over the joint surplus, where  $\eta_t$  is the workers relative bargaining power.

$$w_t \equiv \max \left\{ (H_t)^{\eta_t} (J_t)^{1-\eta_t} \right\}, \quad 0 < \eta_t < 1 \quad (32)$$

The bargaining solution implies

$$\eta_t J_t = (1 - \eta_t) H_t \quad (33)$$

where

$$H_t = \phi_n H_{n^o,t} + (1 - \phi_n) H_{n^r,t} \quad (34)$$

The aggregate worker surplus is given as a weighted average of the surpluses according to their share in the labour force.

The bargaining set, the total surplus, is given by  $S_t = \bar{w}_t - \underline{w}_t$ , where  $\bar{w}_t$  is the maximum wage where firm's surplus  $J_t = 0$ , and  $\underline{w}_t$  is the minimum wage where workers

surplus  $H_t = 0$ . The negotiated wage is the weighted average of these reservation wages,  $w_t = \eta_t \bar{w}_t + (1 - \eta_t) \underline{w}_t$ . Furthermore, the discount factor of firms equals the discount factor of optimizing households, as these own the firms. Thus, we obtain

$$\begin{aligned}
w_t = & \eta_t \frac{1}{\phi_p} p_t^w a_t^n + (1 - \eta_t) w_u + \eta_t \frac{1}{\phi_p} \beta E_t \left\{ \kappa \frac{q_{t+1} v_{t+1}}{n_t} \Lambda_{t,t+1}^o p_{t+1} \right\} \\
& + \eta_t \frac{1}{\phi_p} \beta E_t \left\{ \Lambda_{t,t+1}^o \frac{\kappa}{2} \left( \frac{q_{t+1} v_{t+1}}{n_t} \right)^2 \right\} \\
& + (1 - \eta_t) \beta \frac{1 - \phi_n}{\phi_p} (\rho - p_{t+1}) E_t \{ H_{n^r,t+1} (\Lambda_{t,t+1}^o - \Lambda_{t,t+1}^r) \} \quad (35)
\end{aligned}$$

This equation collapses to a standard equation when workers and firms have equal discount factors, when  $\phi_p = 1$  and when the part relating to the quadratic employment adjustment costs is removed.

The labour market is characterized by three equilibrium equations, (31), (16) and (35).

$$\kappa \frac{q_t v_t}{n_{t-1}} = p_t^w a_t^n - \phi_p w_t + \beta E_t \left[ \Lambda_{t,t+1}^o \kappa \frac{q_{t+1} v_{t+1}}{n_t} \rho + \Lambda_{t,t+1}^o \frac{\kappa}{2} \left[ \frac{q_{t+1} v_{t+1}}{n_t} \right]^2 \right] \quad (36)$$

$$H_{n^r,t} = w_t - w_u + \beta E_t \left[ \Lambda_{t,t+1}^r (\rho - p_{t+1}) H_{n^r,t+1} \right], \quad (37)$$

$$\begin{aligned}
w_t = & \eta_t \frac{1}{\phi_p} p_t^w a_t^n + (1 - \eta_t) w_u + \eta_t \frac{1}{\phi_p} \beta E_t \left\{ \kappa \frac{q_{t+1} v_{t+1}}{n_t} \Lambda_{t,t+1}^o p_{t+1} \right\} \\
& + \eta_t \frac{1}{\phi_p} \beta E_t \left\{ \Lambda_{t,t+1}^o \frac{\kappa}{2} \left( \frac{q_{t+1} v_{t+1}}{n_t} \right)^2 \right\} \\
& + (1 - \eta_t) \beta \frac{1 - \phi_n}{\phi_p} (\rho - p_{t+1}) E_t \{ H_{n^r,t+1} (\Lambda_{t,t+1}^o - \Lambda_{t,t+1}^r) \} \quad (38)
\end{aligned}$$

### 3.6 Intermediate Good Firms

Intermediate good firms purchase homogeneous goods from the wholesale sector and relabel them to produce differentiated goods. These differentiated goods are then sold in a monopolistic competitive market to the final good firms. Furthermore, we assume that intermediate good firms are subject to price stickiness, whereby a fraction  $\chi$  cannot

re-optimize its price in a certain period and set price  $\tilde{P}_t$ . Defining the optimally reset price as  $P^*$ , the aggregate price  $P$  is given by  $P_t = \chi P_{t-1} + (1 - \chi)P_t^*$ . Normalizing, this equation by  $P_t$ , we get:

$$1 = \chi \pi_t^{-1} + (1 - \chi) p_t^* \quad (39)$$

where  $p_t^* = \frac{P_t^*}{P_t}$  is the "real" reset price.

Firms being able to optimize chose price  $p_t^*$  by maximizing their discounted stream of real profits.

$$\max_{P_t^*} E_t \sum_{s=0}^{\infty} (\chi \beta)^s \Lambda_{t,t+s} \left[ \frac{P_t^*}{P_{t+s}} - p_{t+s}^w \right] Y_{i,t+s} \quad (40)$$

subject to the demand equation (48).  $p_t^w$  represents the (real) purchasing price of wholesale goods, and thus the marginal costs.

The first order condition is

$$f_{1,t} = \frac{1}{\mu} f_{2,t} \quad (41)$$

where

$$f_{1,t} = (p_t^*)^{\frac{\mu}{1-\mu}} Y_t p_t^w + \Lambda_{t,t+1} \chi \beta \pi_{t+1}^{\frac{-\mu}{1-\mu} - 1} \left( \frac{P_t^*}{P_{t+1}^*} \right)^{\frac{\mu}{1-\mu}} f_{1,t+1} \quad (42)$$

$$f_{2,t} = (p_t^*)^{\frac{1}{1-\mu}} Y_t + \Lambda_{t,t+1} \chi \beta \pi_{t+1}^{\frac{-\mu}{1-\mu}} \left( \frac{P_t^*}{P_{t+1}^*} \right)^{\frac{1}{1-\mu}} f_{2,t+1} \quad (43)$$

Firms set their price not at the current optimal level but at the level they deem optimal over the expected lifetime of their set price. In the presence of inflation, this means that firms having reset their price earlier will have a lower relative price than firms that just reset their price, and will therefore have a higher share of aggregate demand. This means that marginal products are not equal across firms, but that there will be inefficiencies due to price dispersion, denoted with the symbol  $s_t$ . This means that the quantity available for



aggregate demand,  $y_t$ , is not necessarily equal to the quantity from the per firm production function  $y_t^w$ , but only a fraction  $\frac{1}{s_t}$  of it. Hence, we have the relationships

$$y_t^w = s_t y_t \quad (44)$$

$$s_t = (1 - \chi) \tilde{p}_t^{-\frac{1}{1-\mu}} + \chi \pi_t^{\frac{1}{1-\mu}} s_{t-1} \quad (45)$$

In a zero inflation steady state the optimal reset price will be given by  $\frac{1}{\mu} = p^w$ , where we state again that  $p^w$  are the real marginal cost. Thus, firms set price as a mark-up on real marginal costs.

### 3.7 The Final Goods Firm

The final good,  $Y_t$ , is produced in a competitive market according to the following CES technology:

$$Y_t = \left( \int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{\mu} \quad \mu \geq 1 \quad (46)$$

where each input  $Y_{i,t}$  is a differentiated intermediate good. The term  $\varepsilon = \frac{1}{1-\mu}$  indicates the price elasticity of the demand for any intermediate good  $i$ .

Each period, final goods producers choose a continuum of differentiated intermediate goods,  $Y_{i,t}$ , to maximize their profits subject to the CES technology (46)

$$\max_{Y_{i,t}} \left( \int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{\mu} - \int_0^1 \frac{P_{i,t}}{P_t} Y_{i,t} di$$

where  $\frac{P_{i,t}}{P_t}$  denotes the relative price of the intermediate good  $i$  to the aggregate price index  $P_t$ .

The first order condition with respect to  $Y_{i,t}$  is

$$Y_{i,t}^{\frac{1}{\mu}-1} \left( \int_0^1 Y_{i,t}^{\frac{1}{\mu}} di \right)^{\mu-1} - \frac{P_{i,t}}{P_t} = 0 \quad (47)$$

Using equation (46), one can deduce the intermediate goods demand function from the preceding equation:

$$Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{\frac{\mu}{1-\mu}} Y_t \quad (48)$$

### 3.8 Functional forms

The production function is a CES function using capital and labour. Expressed per household, it reads

$$y_t^w = \left[ \alpha (z_{k,t} B_k k_t)^{\frac{\zeta-1}{\zeta}} + (1-\alpha) (z_{n,t} B_n \phi_p n_t)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}}, \quad (49)$$

where  $\zeta$  is the elasticity of substitution,  $B_k$  and  $B_n$  are technology (scaling) parameters,  $z_k$  and  $z_n$  are capital and labour augmenting productivity shocks and  $\alpha$  is a share parameter.

The Cobb-Douglas case occurs when  $\zeta = 1$ .

The marginal product of labour,  $a_t^n$ , and capital,  $a_t^k$ , are given by

$$a_t^n = (1-\alpha) (z_{n,t} B_n \phi_p)^{\frac{\zeta-1}{\zeta}} \left( \frac{y_t^w}{n_t} \right)^{\frac{1}{\zeta}} \quad (50)$$

$$a_t^k = \alpha (z_{k,t} B_k)^{\frac{\zeta-1}{\zeta}} \left( \frac{y_t^w}{k_t} \right)^{\frac{1}{\zeta}} \quad (51)$$

Capital adjustment costs  $\phi_t$  are quadratic and given by the following function:

$$\phi_t = \frac{\eta_k}{2} \left( \frac{x_{o,t}}{x_{o,t-1}} - 1 \right)^2. \quad (52)$$

with  $\phi_t' = \eta_k \left( \frac{x_t}{x_{t-1}} - 1 \right)$ , and  $\phi = 0$  at the steady state.

The cost of adjusting capacity utilizing is defined as:

$$\mathfrak{S}(u_{k,t}) = \frac{r^k}{\psi} \left( e^{\psi(u_{k,t}-1)} - 1 \right) \quad (53)$$

It follows that the first derivative of cost function with respect to capacity utilization is  $\frac{\partial \mathfrak{S}(u_{k,t})}{\partial u_{k,t}} = r_{k,t} e^{\psi(u_{k,t}-1)}$ . Since  $u_k = 1$ , at the steady state, capacity utilization costs are zero at the steady state  $\mathfrak{S}(u_k) = 0$ .

### 3.9 Monetary policies and resource constraint

Due to the lower zero bound on monetary policy, the interest rate set by the Central Bank is the maximum of the interest rate as determined by a Taylor rule  $R_t^{n*}$  and zero.

$$R_t^n = \max [R_t^{n*}, 0] \quad (54)$$

The Taylor rule set the interest rate according to a criteria of interest rate smoothing, and measures of inflation and output.  $\rho_m$  is the parameter driving the Taylor rule inertia, while  $\phi_\pi$  and  $\phi_y$  are the parameters setting the response of the interest rate to inflation and output.

$$\frac{R_t^{n*}}{R^{n*}} = \left\{ \frac{R_{t-1}^{n*}}{R^{n*}} \right\}^{\rho_m} \left\{ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{y_t}{y} \right)^{\phi_y} \right\}^{1-\rho_m} \quad (55)$$

The resource constraint is given by summing the budget constraint of both type of households eq 12, 19 as well as the profit equation of firms. We here assume that unemployment benefits are financed by lump-sum transfers, which implies that they do not appears in the resource constraint.

$$y_t = c_t + x_t + \frac{\kappa q_t^2 v_t^2}{2 n_{t-1}} + \mathfrak{J}(u_{k,t}) \phi_c k_{o,p,t-1} \quad (56)$$

The experiment conducted in this paper is to consider a decline in the bargaining power of workers, which is assumed to follow an auto-regressive process, with  $\varepsilon_\eta$  a stochastic shock:

$$\eta_t = \rho_\eta \eta + (1 - \rho_\eta) \eta_t + \varepsilon_\eta \quad (57)$$

### 3.10 Equilibrium

The stationary equilibrium consists in processes for the flow variables  $[y, y^w, c, c_o, c_r, x, x_o, a_n]$ , the stock variables  $[n, n_o, n_r, k, k_o, k_{o,p}]$ , the prices  $[R_n, r_k, \phi, u_k, w, p^w, \pi, \tilde{p}, f^1, f^2, s]$ , the labour market rates  $[q, v, p, \theta]$ , the utility and discount rates  $[H_r, \lambda_o, \Lambda^o, \lambda_r, \Lambda^r]$  and the exogenous processes  $[z_k, z_n]$ , given the structural parameters  $[\phi_c, v, \sigma^o, \sigma^r, \beta, \delta, \psi, \eta_k]$ , the labour market parameters  $[\kappa, \rho, \gamma_m, \gamma]$ , the production parameters  $[\alpha, B_k, B_n, \zeta]$ , the pricing parameters  $[\varepsilon, \chi]$  and the policy parameters  $[w_u, \rho_m, \phi_\pi, \phi_y, \xi_\tau]$  satisfying the following equilibrium conditions.

#### Quantities

$$y_t = c_t + x_t + \frac{\kappa q_t^2 v_t^2}{2 n_{t-1}} + \frac{r^k}{\psi} \left( e^{\psi(u_{k,t}-1)} - 1 \right) \phi_c k_{o,p,t-1} \quad (58)$$

$$c_t = \phi_c c_{o,t} + (1 - \phi_c) c_{r,t} \quad (59)$$

$$n_t = \phi_n n_{o,t} + (1 - \phi_n) n_{r,t} \quad (60)$$

$$k_{o,t} = u_{k,t} k_{o,p,t-1} \quad (61)$$

$$k_t = \phi_c k_{o,t} \quad (62)$$

$$x_t = \phi_c x_{o,t} \quad (63)$$

$$k_{o,p,t} = (1 - \delta) k_{o,p,t-1} + x_{o,t} \left( 1 - \frac{\eta_k}{2} \left( \frac{x_{o,t}}{x_{o,t-1}} - 1 \right)^2 \right) \quad (64)$$

$$y_t^w = \left[ \alpha (z_{k,t} B_k k_t)^{\frac{\zeta-1}{\zeta}} + (1 - \alpha) (z_{n,t} B_n \phi_p n_t)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \quad (65)$$

$$a_t^n = (1 - \alpha) (z_{n,t} B_n \phi_p)^{\frac{\zeta-1}{\zeta}} \left( \frac{y_t^w}{n_t} \right)^{\frac{1}{\zeta}} \quad (66)$$

$$r_{k,t} = p_t^w \alpha (z_{k,t} B_k)^{\frac{\zeta-1}{\zeta}} \left( \frac{y_t^w}{k_t} \right)^{\frac{1}{\zeta}} \quad (67)$$

*Household decisions*

$$c_{r,t} = w_t n_{r,t} + w_u (1 - n_{r,t}) \quad (68)$$

$$\lambda_{r,t} = c_{r,t}^{-\sigma^r} \quad (69)$$

$$\Lambda_{t,t+1}^r = \frac{\lambda_{r,t+1}}{\lambda_{r,t}} \quad (70)$$

$$\lambda_{o,t} = c_{o,t}^{-\sigma^o} \quad (71)$$

$$\Lambda_{t,t+1}^o = \frac{\lambda_{o,t+1}}{\lambda_{o,t}} \quad (72)$$

$$\Lambda_{t,t+1}^o = \frac{1}{\beta} \frac{\pi_{t+1}}{R_{n,t}} \quad (73)$$

$$\phi_t = \beta E_t \left( \Lambda_{t,t+1}^o \left[ r_{k,t+1} u_{k,t+1} - \frac{r^k}{\psi} \left( e^{\psi(u_{k,t+1}-1)} - 1 \right) k_{o,p,t} + \phi_{t+1} (1 - \delta) \right] \right) \quad (74)$$

$$\phi_t = \frac{1 - \beta E_t \left\{ \phi_{t+1} \Lambda_{t,t+1}^o \left( \frac{x_{t+1}^o}{x_t^o} \right)^2 \phi'_{t+1} \right\}}{1 - \left( \phi_t + \frac{x_t^o}{x_{t-1}^o} \phi'_t \right)} \quad (75)$$

$$r_{k,t} = r_k e^{\psi(u_{k,t}-1)} \quad (76)$$

*Labour market*

$$n_t = \rho n_{t-1} + q_t v_t \quad (77)$$

$$n_{r,t} = \rho n_{r,t-1} + q_t v_t \quad (78)$$

$$q_t = \gamma_m \theta_t^{-\gamma} \quad (79)$$

$$p_t = \gamma_m \theta_t^{1-\gamma} \quad (80)$$

$$\theta_t = \frac{v_t}{1 - n_{t-1}} \quad (81)$$

$$\kappa \frac{q_t v_t}{n_{t-1}} = p_t^w a_t^n - \phi_p w_t + \beta E_t \left[ \Lambda_{t,t+1}^o \kappa \frac{q_{t+1} v_{t+1}}{n_t} \rho + \Lambda_{t,t+1}^o \frac{\kappa}{2} \left[ \frac{q_{t+1} v_{t+1}}{n_t} \right]^2 \right] \quad (82)$$

$$H_{n^r,t} = w_t - w_u + \beta E_t \left[ \Lambda_{t,t+1}^r (\rho - p_{t+1}) H_{n^r,t+1} \right] \quad (83)$$

$$\begin{aligned} w_t = & \eta_t \frac{1}{\phi_p} p_t^w a_t^n + (1 - \eta_t) w_u + \eta_t \frac{1}{\phi_p} \beta E_t \left\{ \kappa \frac{q_{t+1} v_{t+1}}{n_t} \Lambda_{t,t+1}^o p_{t+1} \right\} \\ & + \eta_t \frac{1}{\phi_p} \beta E_t \left\{ \Lambda_{t,t+1}^o \frac{\kappa}{2} \left( \frac{q_{t+1} v_{t+1}}{n_t} \right)^2 \right\} \\ & + (1 - \eta_t) \beta \frac{1 - \phi_n}{\phi_p} (\rho - p_{t+1}) E_t \{ H_{n^r,t+1} (\Lambda_{t,t+1}^o - \Lambda_{t,t+1}^r) \} \end{aligned} \quad (84)$$

*Price rigidity*

$$f_t^1 = \frac{\varepsilon - 1}{\varepsilon} f_t^2 \quad (85)$$

$$f_t^1 = \tilde{p}_t^{-1-\varepsilon} y_t p_t^w + \beta \chi E_t \left( \Lambda_{t,t+1}^o \pi_{t+1}^\varepsilon \left( \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-1-\varepsilon} f_{t+1}^1 \right) \quad (86)$$

$$f_t^2 = \tilde{p}_t^{-\varepsilon} y_t + \beta \chi E_t \left( \Lambda_{t,t+1}^o \pi_{t+1}^{\varepsilon-1} \left( \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\varepsilon} f_{t+1}^2 \right) \quad (87)$$

$$1 = \chi \pi_t^{\varepsilon-1} + (1 - \chi) \tilde{p}_t^{1-\varepsilon} \quad (88)$$

$$s_t = (1 - \chi) \tilde{p}_t^{-\varepsilon} + \chi \pi_t^\varepsilon s_{t-1} \quad (89)$$

$$y_t^w = s_t y_t \quad (90)$$

*Policy*

$$\frac{R_t^n}{R^n} = \left\{ \frac{R_{t-1}^n}{R^n} \right\}^{\rho_m} \left[ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_y} \right]^{1-\rho_m} e^{\varepsilon_{m,t}} \quad (91)$$

*Exogenous processes*

$$z_{n,t} = z_n^{1-\rho_{zn}} z_{n,t-1}^{\rho_{zn}} e^{\varepsilon_{zn}} \quad (92)$$

$$z_{k,t} = z_k^{1-\rho_{zk}} z_{k,t-1}^{\rho_{zk}} e^{\varepsilon_{zk}} \quad (93)$$

$$\eta_t = \rho_\eta \eta + (1 - \rho_\eta) \eta_t + \varepsilon_\eta \quad (94)$$

## 4 Steady State and Calibration

The steady state of the model described by equations (58) to (94) is given when all variables are constant over time. In principle, the steady state can be solved given all structural parameters. In practice, it is usual procedure to calibrate target values for certain variables and derive structural parameters from these. This section first derives the steady state values of variables, taking into account our calibration strategy. Afterwards, the actual calibration is presented.

### 4.1 Steady State

In a zero inflation steady state with  $\pi = 1$ , we can normalize the price level to unity, and thus obtain  $\tilde{p} = 1$ ,  $s = 1$  and  $p^w = \frac{\varepsilon-1}{\varepsilon}$ . Furthermore, capital utilization is at unity  $u_k = 1$ , while Tobin's  $q$  is unity in the absence of an investment tax or subsidy,  $\varphi = 1 + \tau^x$  and . Therefore, we can derive

$$R_n = \frac{1}{\beta} \quad (95)$$

$$r_k = \left( \frac{1}{\beta} - 1 + \delta \right) \frac{1 + \tau^x}{1 - \tau^k} \quad (96)$$

A usual procedure is to calibrate the job separation rate  $\rho$ , the job finding rate  $p$ , the labour market tightness  $\theta$  and the matching function parameter  $\gamma$ . This allows to derive

$$\gamma_m = \frac{p}{\theta^{1-\gamma}} \quad (97)$$

$$q = \gamma_m \theta^{-\gamma} \quad (98)$$

$$n = \frac{p}{1 - \rho + p} \quad (99)$$

$$v = \theta(1 - n) \quad (100)$$

Since the replacement wage is the same for both types of households, the employment ratio will also be the same in steady state, thus  $n = n_o = n_r$ . The number of optimizing



consumers in employment approaches zero as  $v \rightarrow 0$ .

When using a CES production function, it is useful to normalize output at unity, thus  $y = 1$ . This requires to set the technology parameters multiplying the factors at the inverse of the factor's steady state values.

$$B_k = \frac{r_k}{\alpha p^w} \quad (101)$$

$$B_n = \frac{1}{\phi_p n} \quad (102)$$

$$a^n = (1 - \alpha) (B_n \phi_p)^{\frac{\xi-1}{\xi}} \left( \frac{1}{n} \right)^{\frac{1}{\xi}} \quad (103)$$

Using this procedure,  $\alpha$  actually represents the steady state factor share of capital. One has to keep in mind that changes in model parameters that change steady state employment do not change the factor share distribution in steady state, as the technology parameters are adjusted as well. Furthermore, output per household will always be unity for different participation rates by optimizing households, as we adjust  $B_n$ .

Furthermore, it is useful to define the replacement wage as a fraction  $\omega$  of the steady state after tax wage, thus  $w_u = \omega w$ . Using this equation as well as the steady state versions of the labour market equations (36), and (38), we can numerically derive the steady state values of  $w$ , and the parameter  $\kappa$  required for the labour market to solve to the equilibrium values calibrated above. The system of equations to be solved is given by

$$\kappa \frac{qv}{n} = p^w a^n - \phi_p w + \beta \left[ \kappa \frac{qv}{n} \rho + \frac{\kappa}{2} \left[ \frac{qv}{n} \right]^2 \right] \quad (104)$$

$$\begin{aligned} \phi_p w = & \eta p^w a^n + (1 - \eta) \omega \phi_p w + \eta \beta p \kappa \frac{qv}{n} \\ & + \eta \beta \left\{ \frac{\kappa}{2} \left( \frac{qv}{n} \right)^2 \right\} \end{aligned} \quad (105)$$

The solution for  $\kappa$  will be independent of  $\phi_p$ , while  $w$  will be larger the smaller is  $\phi_p$ . The intuition is that we calibrate output per household to unity, while the labour share in the production function is  $\alpha$ . A smaller labour force participation (as is implied by

smaller  $\phi_p$ ) requires a larger real wage. This will also increase the consumption share of rule-of-thumb households.

We then get the steady state for  $H_{n^r}$

$$H_{n^r} = \frac{(1 - \omega)w}{1 - \beta(\rho - p)} \quad (106)$$

The steady state of aggregate consumption is a residual of the resource constraint:

$$c = y - x - \frac{\kappa q^2 v^2}{2n} \quad (107)$$

Lastly, consumption steady states of both type of households are defined as follow:

$$c_r = wn_r + w_u(1 - n_r) \quad (108)$$

$$c_o = \frac{c - (1 - \phi_c)c_r}{\phi_c} \quad (109)$$

Table 1 shows the parameter calibration used for the numerical simulations carried out further below. The parameters are essentially taken from Gertler and Trigari (2009), who estimated a similar model for the US economy. The relative risk aversion is identical for both households  $\sigma^o = \sigma^r$  and is set at 1. It follows that the utility function takes the form of a logarithmic function. The risk aversion  $\beta$  is set at 0.992, generating an annual interest rate of 3.2%. Capital depreciates at a rate of 2.5% per quarter, which corresponds to 10% annual rate of depreciation. The cost of capital adjustment  $\eta_k$  is 3 and the cost of capacity utilization is set at 0.7 following the estimation made by Sala et al. (2008) for the US economy.

The parameters of the labour market are conventional and taken from Shimer (2005). The job surviving rate  $\rho$  is set at 90%, while the job finding probability  $p$  and the labour market tightness are equal to 0.95 and 0.5 at the steady state respectively. The elasticity of matching to unemployed workers  $\gamma$  is 0.5. An important parameter in search and matching models is the replacement ratio  $\omega$ . In models without strong wage stickiness, a high value is needed to generate realistic employment fluctuations. Gertler and Trigari

(2009) estimate this value to be 0.72 in a model with wage stickiness and 0.98 in a model without wage stickiness. We choose an intermediate value  $\omega = 0.9$ . Since restrictions are placed on two variables  $p$  and  $\theta$ , the steady states for labour market variables are found by solving endogenously for the two parameters  $\gamma_m$ , the efficiency of the matching function, and  $\kappa$ , the employment adjustment cost. They are respectively equal to 1.345 and 0.6572. This parameters produces an employment rate  $n$  of nearly 90% at the steady state.

[Table 1 about here.]

We define the baseline model when optimizing households participate fully in the labour market, thus  $v = 1$ . The share of rule of thumb households ( $1 - \phi_c$ ) is 70%. Furthermore, the production function assumes a lower degree of substitution between labour and capital  $\zeta = 0.4$ , an intermediate value between the Cobb-Douglas case  $\zeta = 1$  and the Leontief case  $\zeta = 0$ . To simulate a standard New Keynesian model, we set the share of optimizing households and the elasticity of substitution between capital and labour to unity ( $\phi_c = 1, \zeta = 1$ ). We also consider another extreme case where optimizing households are identified as capitalists, thus not earning labour income ( $v = 0$ ). All intermediate calibrations are possible.

The elasticity of output to capital  $\alpha$  is set at 0.3 and the mark up is set at 11% generating a labour share of income of 63% at the steady state. The coefficient  $B_k$  and  $B_n$  are equal to 0.122 and 1.1053 respectively, which corresponds to the inverse of the steady state value of the capital stock and employment in order to normalize the CES production function. The set of parameters related to nominal price rigidities is conventional. 75% of firms are unable to adjust their price to the optimal price every period. The demand elasticity  $\varepsilon$  is also fixed at 10.09 to generate a mark up of 11%. Monetary policy inertia  $\rho_m$  is set at 0.8, while the reaction of the interest rate to inflation and output are 1.7 and 0.2 respectively. Lastly, the experiment undertaken consists in a negative shock on the

bargaining power of workers  $\eta$ , which is equal to 0.5 at the steady state. The shock is given by  $\varepsilon = (0.05\eta)^2$  and produces a 1.5% decline in the labour share of income in the baseline calibration. The persistence of the shock  $\rho_\eta$  is 0.9.

## 5 Results

### 5.1 Baseline results

This section presents the simulated results of a fall in worker’s bargaining power using the model described in this paper. The solid line in Figure 2 shows the baseline calibration, while the dashed line shows a standard New Keynesian model with search and matching in the labour market.<sup>6</sup> The figures for output, consumption and investment below represent percentage point deviations in terms of GDP, which in turn is normalized to one in steady state. Inflation, employment and the labour share are represented as percentage point deviations.

[Figure 2 about here.]

The fall in bargaining power causes a fall in the labour share for three reasons. First, workers can only bargain for lower wages, leading to a fall in real wages. Second, employment adjusts slowly with search and matching in the labour market. Third, with low substitution between capital and labour, the price effect from a fall in the wage is not countered fully by the quantity effect from an eventual rise in employment. In the standard New Keynesian model, only the first two effects are present, explaining why the labour share quickly returns to its baseline value. Contrastingly, the labour share falls by more than 1% on impact and stay below steady state for more than 10 quarters in the baseline calibration.

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<sup>6</sup>The baseline calibration includes rule of thumb households  $\phi_c = 0.3$  and full participation of optimizing households in the labour market  $\nu = 1$ . The production function has low degree of substitution between labour and capital  $\zeta = 0.4$ . The standard New Keynesian calibration assumes away rule of thumb households  $\phi_c = 1$ , while optimizing households participate in the labour market  $\nu = 1$ . The production function is Cobb-Douglas  $\zeta = 1$ , while there is no bound on the interest rate.

The experiment of lowering workers bargaining power usually raises output, consumption, investment and employment in a Standard New Keynesian model. Firms' surplus from employment relationships rises, thus increasing vacancies, the number of matches, employment and output. This increases both the marginal product of capital and aggregate saving, thus raising investment. Consumption rises since permanent income increases. Inflation falls since the increase in employment and the fall in wages lowers marginal costs.

The baseline calibration additionally takes several aspects into account that are ignored by the standard New Keynesian model. First, the presence of rule of thumb consumers makes the income distribution an important driver of aggregate demand. This channel is absent in the standard model since a representative household earns all income. Second, a lower elasticity of substitution between capital and labour implies a larger fall in the labour share due to a fall in bargaining power since the price and substitution effect from a fall in wages do not cancel. Third, the baseline simulation allows for the presence of a liquidity trap. The fall in aggregate demand lowers inflation, in response to which the central bank should lower the nominal interest rate. In the presence of a lower bound on the nominal interest rate, the fall in inflation actually raises the real interest rate. This has a threefold effect. Vacancy posting falls since the surplus from a match decreases, investment demand falls, and consumption demand by optimizing households falls as well.

Summarizing, the baseline simulation allows for strong aggregate demand effects in response to a falling bargaining power of workers to occur. The solid line in Figure 2 shows that under such circumstances a fall in workers' bargaining power leads to long-lasting reductions in important macroeconomic variables. Aggregate consumption declines by 1.2 percent on impact. Output declines by 0.6 percent and employment by 0.4 percent. Additionally, investment also falls for 8 quarters. It follows that a reduction

in worker's bargaining power in a situation of economic recession with low interest rates further depresses economic activity.

## 5.2 Minimum wage as a lower bound on wage

Figure 2 has illustrated the importance of labour income for aggregate demand in the proximity of the lower zero bound in monetary policy. The transmission channel going from labour income to aggregate demand modifies the traditional views on minimum wage. In a standard New-Keynesian model, the minimum wage is seen as hampering the downward adjustment in wages. This in turn limits labour demand of firms and amplifies business cycle fluctuations. Contrastingly, in the present model, the minimum wage sets a lower floor on labour income, which sustains consumption and aggregate demand. The direct negative effect of the minimum wage on labour demand is balanced by its positive impact on aggregate demand.

In this section, minimum wage is modelled in a similar way than the lower zero bound in monetary policy. The actual wage is the maximum between the wage generated by the Nash bargaining between workers and firms and the minimum wage.

$$w_t = \max[w_t^*, w_{\min}] \quad (110)$$

with  $w_t$  the actual wage,  $w_t^*$  the wage negotiated between workers and employers in section 3.5 and  $w_{\min}$  the minimum wage.

Figure 3 reproduces the baseline simulation with (dashed line) and without a minimum wage (solid line). The calibration is similar to the baseline model presented in Figure 2. The lower bound on wages is set at 95% of the real wage steady state. The main result is that the minimum wage reduces the size of the recession following a decline in the bargaining power of workers. The minimum wage reduces the drop in output from 0.6 percent to 0.3 percent on impact. The main effect is that the minimum wage re-

duces the drop in the labour share of income from -1 percent to -0.25 percent on impact. It follows that the drop in consumption is more than two times lower than in the absence of a lower bound on wages, sustaining aggregate demand. A secondary effect is related to the adjustment in price. Since inflation declines less in the presence of the minimum wage, the increase in the real interest is more moderate, which is less detrimental to investment and labour demand. The adjustment in the markup also sustains the surplus from an additional match, which affects the hiring decisions of firms.

[Figure 3 about here.]

### 5.3 Sensitivity analysis

This section presents some sensitivity analysis to illustrate the importance of the different transmission channels at work in the baseline calibration. Figure 4 presents the sensitivity analysis concerning the impact of the income distribution as a driver of aggregate demand. The solid line shows the baseline calibration as presented above. The dashed line shows a calibration of limited labour market participation, where optimizing households behave purely as capitalists and do not participate in the labour market. As a result, a larger share of total consumption cannot be smoothed through borrowing.<sup>7</sup> A fall in bargaining power therefore leads to a larger fall in aggregate demand, and consequently to a more severe depression of economic activity. Furthermore, the model is moved closer to an instability region, thus producing a kink in the dynamic path of the variables.<sup>8</sup>

The dashed-dotted line represents the case where there are no rule-of-thumb consumers, thus there is no demand effect from changes in the functional income distribution. However, the case still allows for the economy to be facing a liquidity trap. In this

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<sup>7</sup>The wage share is more or less fixed due to our calibration strategy.

<sup>8</sup>Increasing price stickiness or decreasing  $\zeta$  moves the economy closer to the instability.



case, the increase in aggregate saving described above, combined with the fall in inflation, do not boost investment but cause the lower bound on the nominal interest rate to be binding. Compared to the baseline model, the absence of income distribution effects on aggregate demand diminishes greatly the importance of this lower bound, depressing economic activity by a much smaller amount compared to the other cases.

[Figure 4 about here.]

Figure 5 shows the sensitivity analysis concerning the importance of the liquidity trap, the elasticity of substitution and price stickiness. The solid line shows the baseline model in the absence of a zero bound on the interest rate. The negative demand effect from the loss of worker's bargaining power is short-lived. Although consumption falls on impact and stays below zero for 4 quarters, output increases. The rise in output is linked with the fall in the nominal interest rate set by the Central Bank. In the absence of a liquidity trap, nominal interest rate falls with inflation leading to a decline in real interest rate. It follows that both labour demand and investment react positively. Employment and investment increase on impact. The speed at which output recovers is partly determined by the existence of capital adjustment costs, which delays investment decisions. Monetary policy shortens the recession by stimulating both supply and demand channels. When all labour income is earned by rule-of-thumb consumers ( $v = 0$ ), then the initial fall in output and consumption is more pronounced and longer.

[Figure 5 about here.]

The dashed line shows the baseline calibration in the absence of zero bounds and in the case of Cobb-Douglas production function. The distinction between Leontief and Cobb-Douglas production function is a central element of the literature discussing changes in the labour share. Following a change in wages, the degree to which firms substitute capital and labour affects the level of employment. In our case, the labour share

adjusts slightly faster for an elasticity of substitution of 1 rather than 0.4. The impact on output and employment is however quasi null, given the strength of the other transmission channels.

Finally, the fall in aggregate consumption demand due to lower income of rule of thumb households relies on the presence of price stickiness. In a flexible price model (dashed point line), only the supply side effects are at work, meaning that the fall in worker's bargaining power raises labour demand, employment and output. Due to the increase in permanent income aggregate consumption also rises.

Figure 6 proves that the mechanisms introduced by this paper, the importance of the income of workers to support aggregate demand, will induce a fall in output and employment after a fall in bargaining power even in the absence of a liquidity trap. The calibration has been changed to have only rule of thumb consumers working, meaning  $v = 0$ , the elasticity of substitution is very low,  $\zeta = 0.05$ , and the average duration of price changes is 8 quarters instead of 4 quarters. Thus, it presents a much more rigid scenario with little substitution between capital and labour and rigid prices.

The solid line in Figure 6 shows that output and employment fall for 3 quarters. Due to the low elasticity of substitution, the fall in bargaining power has a strong effect on the labour share, which, coupled with a strong degree of price stickiness and the impossibility of consumption smoothing, induces a strong negative demand effect. Nevertheless, the fall of the real interest rate combined with the saving shock increase investment, which causes a rise in output after 4 periods. The kink in the solid line shows that this calibration moves the model closer to an instability region.

[Figure 6 about here.]

The dashed line in Figure 6 shows that a lower bound on the real wage also works in the absence of a liquidity trap. Income and consumption of rule of thumb consumers is

supported, which stabilizes aggregate demand and lessens the negative impact of the fall in bargaining power.

## 6 Conclusion

The model presented in this paper shows that under certain parameters a change in income distribution in favour of capital lead to lower employment and output. This result contrasts the conclusion from a standard New Keynesian model, which finds virtue to wage moderation. To reach this result, the modelling strategy has been to reinforce the transmission channel from income distribution to consumption decisions by combining rule-of-thumb households and nominal price rigidities. This transmission is strengthened in the presence of a lower zero bound in monetary policy.

[Table 2 about here.]

Table 5.3 gives an overview of the effects of lower bargaining power on consumption, investment, aggregate demand and labour demand in the present model and in the standard New-Keynesian model. In the New-Keynesian model, the main transmission channel goes through labour demand. The increase in consumption and investment follows from the increase in employment and permanent income. Contrastingly, in the present model, consumption drops due to the presence of rule-of-thumb households, while investment is negatively affected by the increase in the real interest rate. The lower zero bound also affects labour demand negatively. It follows that lower bargaining power leads to lower output and employment in the presence of liquidity trap. An important implication of this model is that downward wage rigidity sustains aggregate demand and reduces the fall in output.

Two extensions to the present paper can be envisioned. The first is to allow workers to have some access to financial markets, and thus engage in some limited borrowing. This allows the study of the effect of inequality on household indebtedness, thereby following Kumhof and Ranciere (2010). Second, the extension to a two country model allows

to study a number of research questions present on the current political agenda. In an open economy, a falling wage will additionally raise export demand, depending on the exchange rate regime. However, such a policy could be a beggar-thy-neighbour policy by raising unemployment in the foreign country. Furthermore, international imbalances might result. Given the results obtained in this paper, an interesting addition to the policy debate is likely to result from these extensions.

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# Figures

Figure 1: (adjusted) labour share of income (at factor cost)

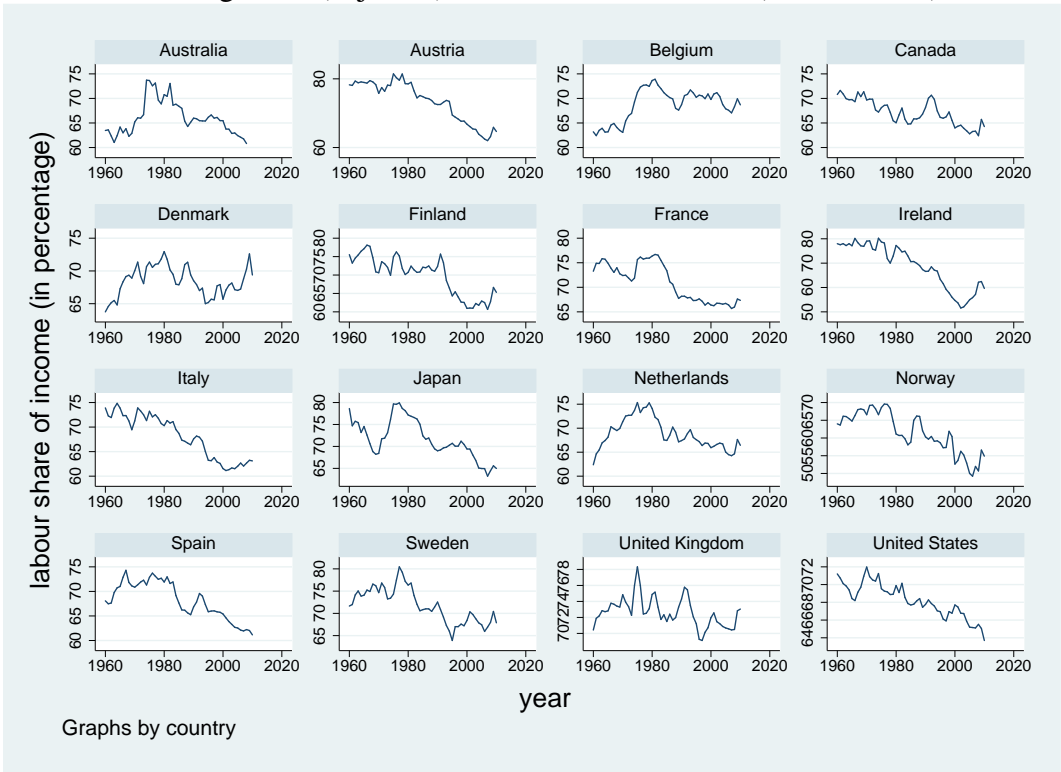




Figure 2: Standard New Keynesian vs baseline model



Figure 3: Minimum wage

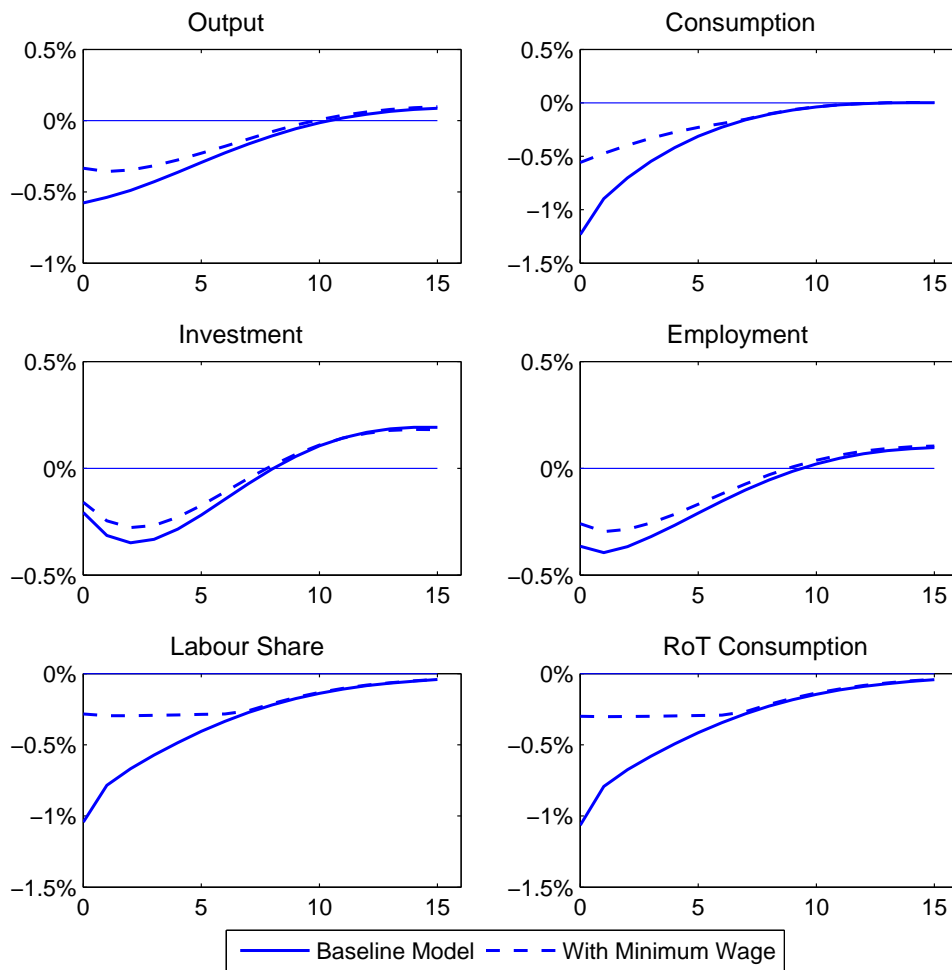


Figure 4: Sensitivity Analysis: Income Distribution

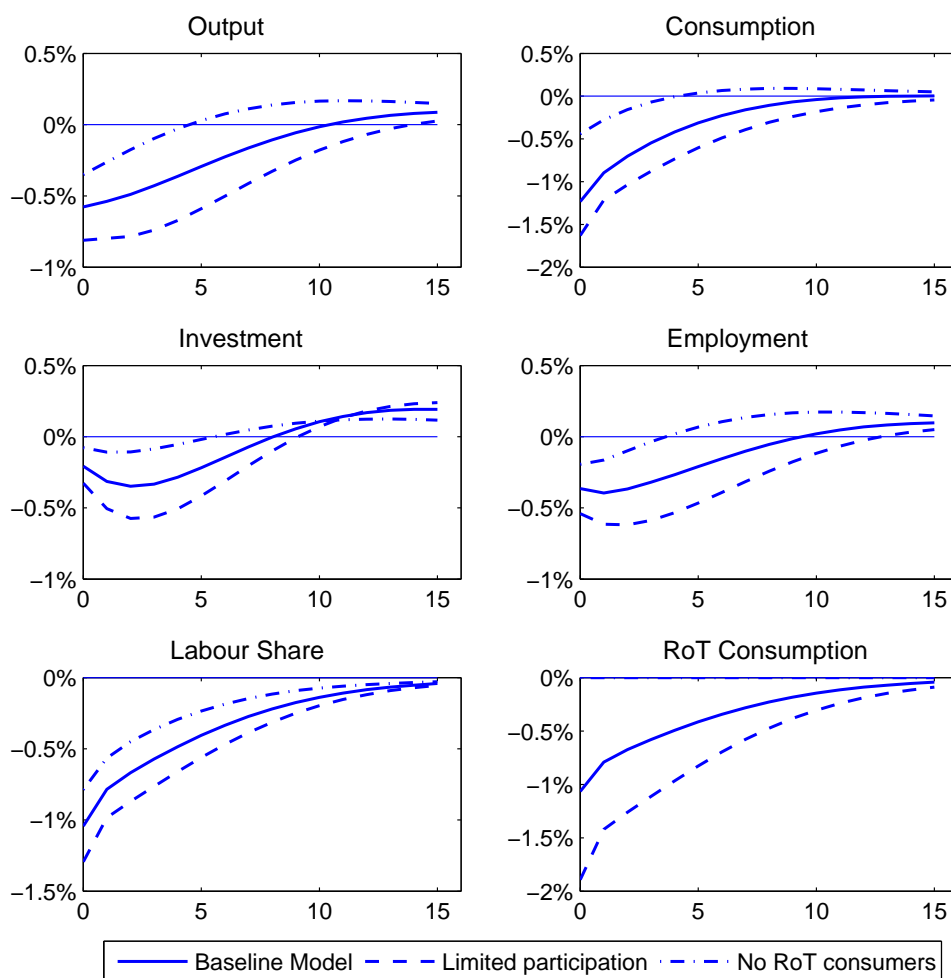


Figure 5: Sensitivity Analysis: CES and Price Stickiness

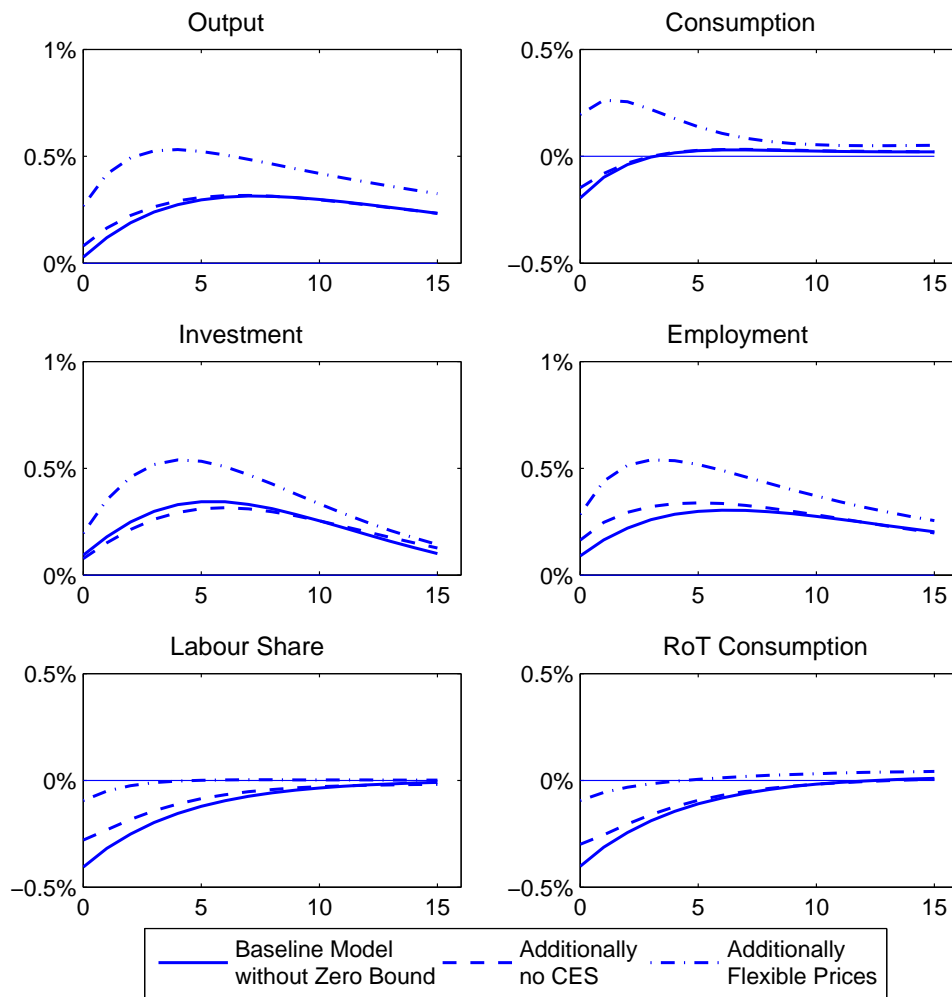
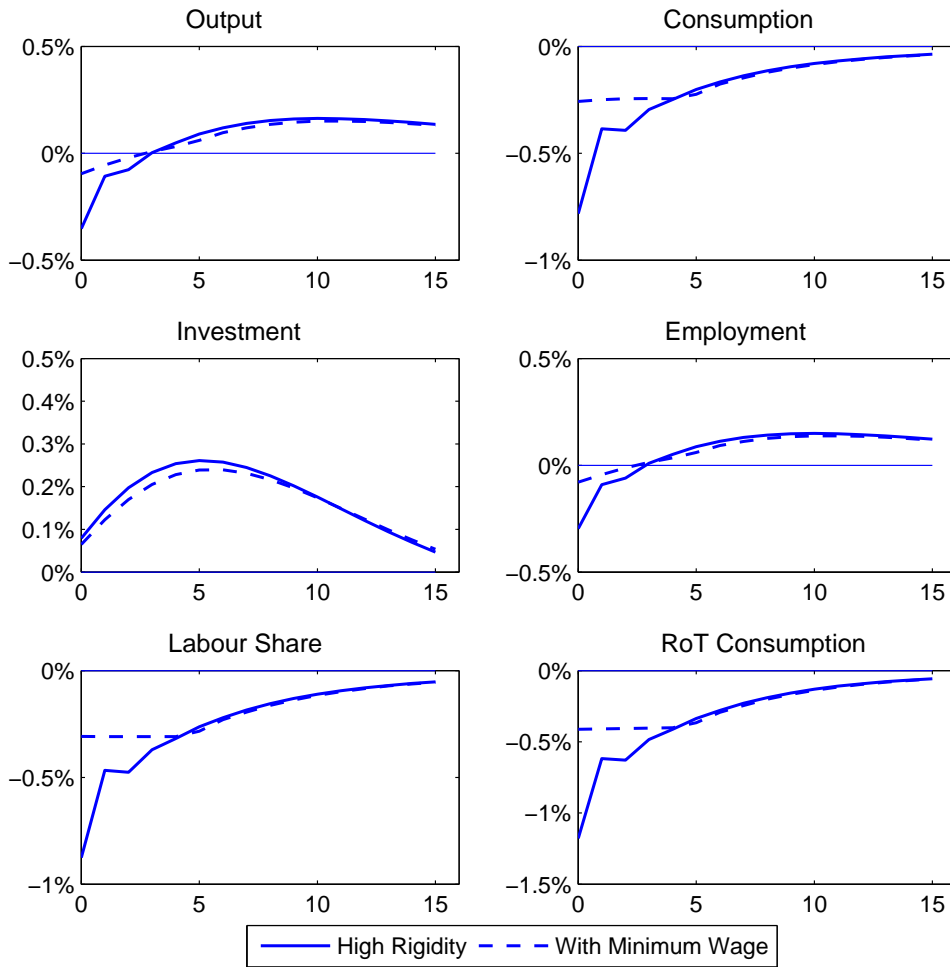


Figure 6: The High-Rigidity Scenario



## Tables

Table 1: Calibration

<i>Structural parameters</i>	
Share of Optimizing Consumers	$\phi_c = 0.3$
Labour market participation of optimizing consumers	$v = 1$
Relative risk aversion parameters	$\sigma^o = \sigma^r = 1$
Discount factor	$\beta = 0.992$
Capital depreciation rate	$\delta = 0.025$
Capital adjustment cost	$\eta_k = 3$
Capital utilization cost	$\psi = 0.7$
<i>Labour market parameters</i>	
Exogenous job loss probability	$1 - \rho = 0.1$
Target job finding probability	$p = 0.95$
Labour market tightness	$\theta = 0.5$
Matching elasticity	$\gamma = 0.5$
Implied matching function parameter	$\gamma_m = 1.345$
Implied employment adjustment cost	$\kappa = 0.6572$
Implied employment rate	$n = 0.9048$
<i>Production parameters</i>	
Capital share	$\alpha = 0.3$
Elasticity of substitution	$\zeta = 0.4$
Capital technology	$B_k = 0.122$
Labour technology	$B_n = 1.1053$
<i>Pricing parameters</i>	
Demand elasticity	$\varepsilon = 10.09$
Price stickiness	$\chi = 0.75$
<i>Policy parameters</i>	
Replacement rate	$\omega = 0.9$
Interest rate smoothing	$\rho_m = 0.8$
Inflation response	$\phi_\pi = 1.7$
Output response	$\phi_y = 0.2$
Bargaining power	$\eta = 0.5$
Bargaining power auto-regressive coefficient	$\rho_\eta = 0.9$

Table 2: Summary of results

Variable	New Keynesian	Baseline
C	+	-
	Permanent inc	RoT
I	+	-
	Employment	lzb
AD	+	-
Labour D	+	-
	wage	lzb