Optimization of the expenditure rule's parameters values by the genetic algorithm

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May 22, 2012

Abstract

Fiscal rules are one of the tools aimed at preventing excessive public debt accumulation. Several government documents announce the forthcoming introduction of the permanent fiscal rule for Poland, the aim of which is the general government balance on the MTO level, -1% GDP in the medium term. In the following article we take an innovative approach to optimizing the parameters of the expenditure rule by the genetic algorithm.

This article presents the detailed proposition of the permanent expenditure rule for Poland. Then the scheme of the implemented genetic algorithm is described. The sensitivity analysis of the algorithm's parameters is also documented. The article concludes with the description of the obtained results.

keywords: expenditure rule, correction mechanism, general government balance, genetic algorithm, optimization of parameters
JEL codes: C61, E62

1 Introduction

While writing this article, the topic of dangerously high debt level, especially in the peripheral eurozone countries (Greece, Ireland, Portugal, Spain, Italy) but also in the USA and even in Poland has been taking up in the media continuously. At the same time, fiscal rules, one of the most important tools aimed at reducing a debt toward safer levels and then for stabilising it, have been gaining in popularity.

Most of the European Union member states, including Poland, adopted the Euro+ Pact (Conclusions of the Heads of State or Government of the euro area, 2011). In this document (p. 10) countries obliged themselves 'to reflect the EU fiscal rules established in the Stability and Growth Pact in the national law'. The choice of a legal act, as well

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as the form of the rule remains in competencies of the countries but it will have to lead to budgetary discipline in the central and local sectors. The rules will be also assessed by the European Commission. However, the EU did not rest there. In December 2011, a package of five regulations and one directive strengthening economic governance in the EU, the so called 'six-pack' entered into force. The abovementioned directive on requirements for budgetary frameworks of the Member States (Council Directive 2011/85/EU, November 2011) stresses that strong numerical fiscal rules, based on the independent analysis, should have clearly established targets and mechanisms of effective and timely monitoring. The member states should besides define escape clauses and consequences of disobeying the rules. Finally, in March 2012 the Treaty on stability, coordination and governance in the EMU (the so called 'fiscal compact') has been signed up. It details requirements concerning fiscal rules implemented into national law in the countries which will sign this document.

There are several fiscal rules applied in Poland. Although the debt limits generally improve fiscal discipline, they can also trigger procyclical actions (see UCP 2011, p. 32). Another, the expenditure rule, which limits growth of some types of expenditure to the CPI index plus 1 pp, has too little coverage to have a crucial impact on the state of public finances. Moreover, this rule is temporary and will be in force as long as excessive deficit procedure will be imposed on Poland.

In the Convergence Programme 2011 not only were disadvantages of two main fiscal rules mentioned (p. 30) but also symptoms of the fiscal imbalance were described (p. 10). Introduction of the new permanent expenditure rule can be useful for three reasons: for achieving fiscal space, which would be necessary during a possible recession, for stabilising medium-term expenditure growth alongside with income growth and for diminishing cyclical fluctuations of economy. The rule would come into effect after cancelling EDP for Poland which is expected to happen in 2013. Its form was briefly outlined in the Multiannual Financial Plan for Poland (p. 13). According to this document, the tangible target of the rule will be a structural balance on the level of the MTO (medium-term objective). The MTO constitutes the minimum required by the European Commission for members of the eurozone and candidate countries (-1% GDP for Poland). However, its formula and precise parameter values are still undefined.

The main aim of the analysis described in this article is to choose the optimal parameter values of a universal expenditure rule. By the same token, the obtained conclusions can contribute to designing the permanent expenditure rule for Poland.

There are two fundamental methods of looking for the best solutions in economic policy, including fiscal rules issues. The first consists in analysing regulations applied currently and in the past in various countries. The second takes advantage of simulations and modelling analyses of scenarios implemented into computer programmes. Each method has its merits and demerits. Conclusions drawn from models can turn out to be inconsistent with reality as a consequence of many assumptions and simplifications. Furthermore, not every solution can be applied in practice. Finally, it is hardly possible to analyse some aspects of a rule, especially those institutional. On the other hand, models offer a possibility to verify ideas which have not been implemented yet.

The article is organised as follows. Chapter 2 explains the proposed formula of the expenditure rule. In chapter 3 the concept of the genetic algorithm optimising the rule's parameter values is outlined. It also includes the sensitivity analysis of algorithm's parameters affecting its effectiveness. Chapter 4 presents the results of the optimisation, while the last chapter consists of a summary and conclusions for fiscal policy.

2 Draft of the new rule

The formula of the rule whose parameter values are optimised in this paper, is one of the preliminary propositions analysed in the Ministry of Finance. However, the proposed formula was slightly modified for the sake of modelling. The formula, composed of five equations described in this chapter, is designed in a such universal way, that it could be used in any country aiming for stabilisation the ratio of a general government balance to GDP.

The main formula [1] implies approximately that expenditures grow as fast as inflation times an average real GDP growth. The expenditure limit for the year t+1 ($L\_n\_exp_{t+1}$) is equal to the limit for the year t adjusted ex post for errors in inflation forecasts and multiplied by the forecasted inflation for the year t+1 ($E_t\{CPI_{t+1}\}$) and by the indicator of a medium term real GDP growth ($Ind(r)_{t+1}$). If a critical cumulation of deviations of a nominal balance from the MTO in the year t+1 is registered, then a correction of a real part of the formula ($C(r)_{t+1}$) is applied. The expenditure limit is assumed to be determined at the beginning of the year t.

$$L\_n\_exp_{t+1} = \overbrace{L\_n\_exp_t^* \cdot \underbrace{E_t\{CPI_{t+1}\}}_{\text{nominal component}} \left[\underbrace{Ind(r)_{t+1}}_{\text{real component}} - \underbrace{C(r)_{t+1}}_{\text{real corrective component}} \right]}^{\text{permanent expenditure rule}} \tag{1}$$

Equation [2] explains how the ex post correction works. There is information at the beginning of the year t about the actual inflation in the year t-1 (CPI_{t-1}). This information can be used in order to revise the limit under- or overvalued due to an erroneus forecast of inflation for the year t-1, made at the beginning of the year t-1 ($E_{t-1}\{CPI_{t-1}\}$). This is reflected by the second fraction in formula [2] ($\frac{CPI_{t-1}}{E_{t-1}\{CPI_{t-1}\}}$). Besides, at the beginning of the year t, thanks to inflow of new pieces of information, the inflation for the year t ($E_t\{CPI_t\}$) can be prepared easier than a year before ($E_{t-1}\{CPI_t\}$), which is reflected by the first fraction in the formula [2] ($\frac{E_t\{CPI_t\}}{E_{t-1}\{CPI_t\}}$).

$$L\_n\_exp_t^* = L\_n\_exp_t \cdot \underbrace{\frac{E_t\{CPI_t\}}{E_{t-1}\{CPI_t\}} \cdot \frac{CPI_{t-1}}{E_{t-1}\{CPI_{t-1}\}}}_{\text{corrective nominal factor}} \tag{2}$$

In equation [3] the indicator of a medium-term real GDP growth ($Ind(r)_{t+1}$) is developed as a geometric mean of real GDP dynamics ($GDPr\_d_T$) noted in the past or forecasted at the beginning of the year t for further years. The parameters p and f stand for a number of respectively past and future years used in the indicator.

$$Ind(r)_{t+1} = \sqrt[p+1+f]{\prod_{T=t+1-p}^{t+1+f} GDPr\_d_T} \tag{3}$$

If it is desirable for the rule to stabilise in the medium term a GG balance on the level of the MTO, a mechanism correcting potential deviations is necessary. Such deviations can result either from the fact, that the nominal GDP elasticity of income differs from 1 or the wrong forecasts of GDP dynamics. Therefore, the proposed rule is combined with the notional account (NA) [4], which amounts to cumulated differences between an actual nominal balance expressed in percentage points (Bn_t) and MTO.

$$NA_t = NA_{t-1} + Bn_t - MTO \tag{4}$$

If the negative value registered on the NA exceeds one of the limits (thresholds): L_-^j, $j = 1, 2$, then a real corrective component of the formula [1] $C(r)_{t+1}$ amounts to β_-^j percentage points. This correction can take place only during sufficiently good times in order not to induce procyclical fiscal policy. Whether times (economic situation) are good or bad is assessed on the basis of a comparison of a forecasted GDP dynamics with the indicator of a medium-term real GDP growth. The difference between $Ind(r)_{t+1}$ and a forecasted GDP dynamics for the year t+1 is sufficient if it exceeds the value α_-^j. Exceeding of the negative threshold NA_-^1 does not imply a correction, unless the correction path is already active. The correction path applies from time when exceeding of L_-^2 is stated until a moment at which the NA comes back below (i. e. closer to zero) L_-^i.

Breaching the positive thresholds by a surplus cumulated on the NA, triggers, consistently with the form of the optimised rule, analogous consequences as the negative thresholds with one exception. Namely, it was assumed that the correction path is not in force on the positive side. However, it should be made clear that, according to the MoF proposal, breaching the positive thresholds would not trigger any automatic adjustments.

$$C(r)_{t+1} = \begin{cases} \beta_+^2, & NA_{t-1} \geq L_+^2 \text{ and } GDPr\_d < Ind(r)_{t+1} + \alpha_+^2 \\ \beta_+^1, & L_+^2 > NA_{t-1} \geq L_+^1 \text{ and } GDPr\_d < Ind(r)_{t+1} + \alpha_+^1 \\ 0, & L_+^1 > NA_{t-1} \geq L_-^1 \\ 0, & L_-^1 > NA_{t-1} \geq L_-^2 \text{ and no correction path} \\ \beta_-^1, & L_-^1 > NA_{t-1} \geq L_-^2 \text{ and correction path and } GDPr\_d > Ind(r)_{t+1} + \alpha_+^1 \\ \beta_-^2, & NA_{t-1} < L_-^2 \text{ and } GDPr\_d > Ind(r)_{t+1} + \alpha_+^2 \end{cases} \tag{5}$$

The alternative rule taken into consideration was a rule determining an annual structural balance - similar to currently being introduced in Germany and recommended in the fiscal compact (Treaty on Stability, Coordination and Governance in the Economic and Monetary Union, 2012). However, it needs applying abstract concepts, such as: potential GDP or an output gap whose values are hardly precisely forecasted and which are revised even after a few years. The additional difficulty in calculating them for Poland are short time series. Hence, the advantage of the formula described in this chapter is that it does not involve calculating an output gap.

3 Optimisation of the parameter values of the expenditure rule

3.1 Method of assessing the rule

The aim of this article is to optimise the parameter values of the rule presented in the previous chapter. The rule includes fourteen parameters: two referring to the indicator of a medium-term real GDP growth (p, f) and four parameters per three groups: the NA limits (thresholds, L), definitions of bad/good times (α), as well as values determining reduction or enlargement of expenditure (β). Each of randomly generated parameters can take on value from a given range.

An assessment of the rule's effectiveness results from a willingness to minimalise two sources of risk. The fiscal compact puts emphasis on fulfiling the MTO every year. Violating this regulation constitutes the first risk from which the effective rule should prevent. Therefore, the first component appearing in the goal function is a mean deviation of a structural balance from the MTO over all 91 years during which the rule was applied.

The second important source of risk in case of an expenditure or a structural balance rule is a possibility of exceeding the 3% threshold by a nominal balance. According to the, so called, 'six-pack' this can imply even financial sanctions imposed on a country by the European Commission (when a warned country uses further a rule which will not allow it for a reduction of a deficit). For that reason, the goal function was supplemented by: a) a component penalising for exceeding 3% GDP by a deficit and amounting to 0.02 per year and b) a square of a difference between a nominal deficit and the 3%-GDP-threshold, taking into account only in case of exceeding this threshold.

Any rule should yield desired effects regardless of an assumed macroeconomic scenario. This is why the ultimate rate of a given set of the parameter values, i. e. the formula for the goal function, is defined as a mean rate over 100 scenarios. Each scenario lasts 100 years and is based on a three-equation new keynesian model (see subchapter 3.4).

The indicator should not consist of more than ca. 10 years because it is designed for estimating a medium-, not a long-term GDP growth. Therefore, the parameters p and f can take on values only from 0 to 5. Furthermore, the sets containing possible values for the parameters: α and β were specified as: 0 pp, 0.5 pp, 1 pp, ..., 5 pp. Only this yields nearly $8 \cdot 10^9$ possible combinations. As an example, if possible threshold values were drawn from the set: 0%, 1%, 2%, ..., 100%, then the number of combinations would equal as many as ca. $2 \cdot 10^{17}$. A limited computing power of processors makes an assessment of all combinations of parameters at the same time and, consequently a choice of the optimal set, virtually impossible. Classical methods searching for an extreme of a function are also inadequate for solving this sort of a problem. This is because the goal function can have a lot of local extremes. In this connection, a genetic algorithm was applied.

3.2 Genesis of a genetic algorithm

Genetic algorithms are one of nonclassical optimisation methods, which rebuffs unfavourable combinations of parameters and, as time goes by, finds better and better solutions thanks to: selection, crossing and mutation. However, it does not ensure achievement of an optimum. Genetic algorithms belong to a class of evolutionary algorithms, which immitate processes occuring in the nature. A human being, which is obviously a very complex mechanism, is the result of evolution. We may venture a conlusion that a fiscal rule is also quite complicated mechanism which shoud be as effective in various macroeconomic conditions. So we can design its optimal form by applying a proper genetic algorithm immitating a mechanism of evolution.

Professor John Henry Holland, the author of a published in 1975 breakthrough article about adaptations in natural and artificial systems is considered to be the father of genetic algorithms. In this paper Holland compares genetics not only with artificial intelligence but also with economic planning, control theory, physiological psychology and game theory. The analogies for chromosomes, which contain genes, are following structures: computer programmes, policies, groupings of cells and strategies. Whereas the analogy for genetic operators, i. e. mutation, recombination and crossing, are respectively: learning rules, productive actions, bayesian rules, modifications of synapses or rules regarding iterative approximation of an optimal strategy. Similarly, as adaptation to given environment determines a supremacy of one organism (solution) above others in the field of genetics, so do: effectiveness in artificial intelligence, utility in economic planning, error functions in control theory, behaviour assessment in physiological psychology or pay-off in game theory.

There can be much more applications of genetic algorithms. Thanks to them it is possible to choose explaining variables into a model predicting a stock exchange index (finance), to forecast demand (economics), to plan a schedule in a production system (management), to reduce noise (physics), to optimise an electroenergetic distributive network (electronics), to colour graphs (graphics) and even to generate a human face (biotechnology). These are only a few examples of applications which can be found on the internet.

3.3 Scheme of the algorithm optimising the rule's parameters

The implemented algorithm is outlined in a pseudocode below (real computation was proceeded in Matlab). As it has been highlighted before, a rule is identified here with an organism. A group of rules in a given iteration is definied as a generation, while a parameter is a trait of an organism transmitted to its offsprings in a form of a gene. A mutation is reflected by a random change of a parameter value. Finally, rules whose parameters are transmitted to a next generation are called 'parents' or to better visualise: 'father' and 'mother'. This algorithm allows for the same organism to play role of both father and mother, so it permits cloning. Another departure from the real process is enabling 1/3 of a generation to be transferred to a next generation. Consequently, the best rules can 'live forever'.

It should be explained beforehand that the function 'draw(0,1)' draws a number from the uniform distribution specified over a range between 0 and 1. If this random number turns out to be lower than a previously declared parameter 'mutation_scale' then a mutation takes place. A mutation of a gene consists in an increase or a decrease (with the same probability) in its value by a unit. Whereas the function 'assess(organism)' evaluates a given set of parameters according to the rules presented in the subchapter 3.1.

```
#genetic algorithm

#initiatlisation of a first generation, assignment random values to genes
    for organism=1 to size_of_generation
        for gene=1 to number_of_genes
            value(gene, organism)=draw(set_of_possible_values(gene))
        next gene
    next organism

#start of a main loop
from iteration=1 to number_of_iterations

    #individual assessment of organisms of a generation
    from organism=1 to size_of_a_generation
        assess(organism)
    next organism

    sorting assessments, determining a tercile and a minimum assessment

    #transmiting the best 1/3 of a generation n to a generation n+1
    from organism=1 to size_of_a_generation
        if assess(organism) < tercile_of_assessments then an organism is transferred to a generation
        from a next iteration, as well as to a set of potential parents and is saved into the file

        #possible mutations
        from gene=1 to number_of_genes
            if draw(0,1) < mutation_scale_1 then value(gene, organism)=new_value(gene, organism)
        next gene

    next organism

    #generating of a random 1/3 of a generation n+1
    from organism=1 to 1/3 · size_of_a_generation
        from gene=1 to number_of_genes
            value(gene, organism)=draw(set_of_possible_values(gene))
        next gene
        organism is transferred to a next generation
    next organism
```

```
    #generating 1/3 of a generation n+1 as offsprings of parents from a generation n
    from organism=1 to 1/3 · size_of_a_generation

        #association of organisms into pairs
        father=draw(1, size_of_a_generation/3)
        mother=draw(1, size_of_a_generation/3)

        #succession of genes
        from gene=1 to number_of_genes
            value(gene, organism)=value(gene, draw(father or mother))

        #possible mutations
        if draw(0, 1) < mutation_scale_2 then value(gene,organism)= new_value(gene, organism)
        next gene

        an organism is transferred to a generation from a next iteration
    next organism

next iteration
```

3.4 Description of the applied new keynesian model

The scenarios which constitute an assessment of a rule were generated in two steps. In the first step, the potential GDP growth path (from 3% in the first year to 0% last year plus/minus random deviations) was established. Lack of an impact of fiscal policy on a long-run (potential) ouput was then assumed.

Nevertheless, we took into account interaction between fiscal policy and GDP in the short run. It was expressed by the econometric, estimated by the generalised method of moments, three-equation new keynesian model which belongs to the mainstream in the literature. Equation [1] makes an inflation rate conditional on an output gap (the positive sign of the estimated parameter is consistent with the concept of the Phillips curve) and on inflation lagged by one year. In equation [2] an output gap is explained by the same lagged variable, an interest rate and a structural balance. The negative impact of both: an interest rate and a balance is consistent with the theory. A structural balance is calculated as an adjusted for cyclical oscillations (expressed in terms of an output gap) difference between expenditures determined by the rule and incomes determined by the GDP growth (so also dependant on an output gap). As a relationship between a gap and a balance is nonlinear, calculations for a single year were made iteratively - till values of macro-fiscal variables were relatively stable. More about the assumptions made in this fragment of the code can be found in subchapter 3.5. Finally, equation [3] stands for the version of the Taylor's rule, in which an output gap and lagged inflation influence an interest rate level. An interest rate is treated here as an average debt interest paid by the GG sector.

The data are for the years 1956-2008 and although they describe the U. S. economy, use of them in the context of Poland is justified. The Polish economy is lacking of sufficiently long time series. Furthermore, they concern mainly the transition period. Consequently, a lot of structural breakings can be then expected. With a great dose of probability, in the horizon of the simulation, which comes to a hundred years, the Polish economy will behave in a similar manner to developed economies, such as the USA. At last, the formula of the rule is so universal that its advantage could be taken of in any country, not only in Poland.

$$\pi_t = \underset{(0.133)}{0.280} + \underset{(0.041)}{0.923}\pi_{t-1} + \underset{(0.087)}{0.390}y_t^*, \qquad R^2 = 0.807, se = 1.032 \qquad [1]$$
$$y_t^* = \underset{(0.056)}{0.601}y_{t-1}^* - \underset{(0.046)}{0.066}i_t - \underset{(0.121)}{0.218}cab_t, \qquad R^2 = 0.363, se = 1.620 \qquad [2]$$
$$i_t = \underset{(0.045)}{0.784} + \underset{(0.073)}{0.308}\pi_{t-1} + \underset{(0.135)}{0.179}y_t^*, \qquad R^2 = 0.794, se = 1.485 \qquad [3]$$

Where: π - inflation approximated as a GDP deflator,
y^* - output gap (real GDP calculated by π and nominal GDP and then adjusted by the Hodrick-Prescott Filter, λ=100),
i - interest rate represented by an effective anual FED Funds rate,
cab - structural balance / GDP, represented by a cyclically adjusted balance of Federal Government,
R^2 - determination coefficient,
se - standard error of regression.

Parameter estimate errors were shown in the parentheses. The data come from databases: U. S. Department of Commerce: Bureau of Economic Analysis, the websites of US Government Spending, Congressional Budget Office and Federal Reserve.

Apart from the above described equations, values of all variables were calculated for every year according to non-estimated equations. Expenditure level was implied by the rule, while income - by an assumption of a unit GDP growth elasticity. A level of interest and an output gap are essential for extracting cylical expenses, whereas cylical expenses - for calculating a structural balance.

In the second step, for every scenario three shocks (distortions), i. e. realisations of random variables drawn from distributions N(0,σ) were randomly generated and added to the variables (inflation, an output gap and an interest rate) explained in equations 1-3. Values of σ come from variance of regression error estimates of the respective equations. These shocks are interpreted typically as: demand, supply and monetary policy shocks. Every rule was analised jointly under all those scenarios.

3.5 Parameter values optimisation of the genetic algorithm

In this subchapter we investigate how sensitive the results are on values of the genetic algorithm's parameters and on different scenarios. This analysis aimed at choosing the best (fast but also effectively minimising a goal function) formula for the algorithm. In addition, it aimed at establishing which factors have the biggest impact on the final results. On the one hand, the set of parameters taken into consideration consists of: parameters determining accuracy of solutions, a method of mutations, a formula for a goal function. On the other hand, a form of a scenario depends on an assumed path of potential GDP growth. In order to avoid the excessive computational complexity, the analysis was relatively simple and did not exhaust the whole issue of optimisation of the algorithm. Contrary to the final results, described in chapter 4, the sensitivity analysis was carried out for the simpler formula of the goal function. It only minimised average deviations of a structural balance from the MTO, without any components penalising exceedance of the threshold of 3% GDP by a deficit. Moreover, values of mutated genes were drawn from a whole possible range rather than randomly modified by a unit upwards or downwards.

Two first parameters taken into investigation were: a maximal number of iterations for one loop referring to a given year and a maximally allowed relative difference between two values of the same variables from succesive iterations. These two parameters, standing for the convergence criterium for one year, determine speed and accuracy of the algorithm.

A maximal number of iterations should be reached as seldom as possible. Its reaching means that in that year no solution of an equation system, consisted of three equations coming from the NKM and the equations for the remaining macro-fiscal variables, could be found. Unless the conditions are rigorous (a small maximally allowed relative difference between variables), the looped procees designed for a one year typically ends after a few iterations. This is the reason why the increment of average time of computation compared with a maximal number of iterations is rather little in the first rows of table 1. It increases as conditions become tougher. As a result, a high maximal number of iterations should be set, because, thanks to a relatively small loss of time, a number of years without a converged solution can be significantly reduced (see eg. the first and second row in table 1 and analogous rows in table 2).

A relative difference between variables is calculated as a maximum of absolute differences between values of variables in successive iterations divided by a standard deviation of a given variable. The variables taken into consideration are as follows: a deflator, an interest rate, an output gap, a structural balance/GDP, a debt/GDP and an expenditures dynamics. Specifying a value for this parameter is an arbitrary question, but we may assume that if differences of all variables get decreased under 1%, then it shows that the algorithm achieved a solution. Hence, a maximal number of iterations for one year was set on the level of 10^5, while an allowed difference was set at 10^{-2}. This implies on average nearly 30 sec. for calculating a solution for a given formula of the rule and 100 scenarios. However, if average oscillations of a structural balance exceed 10 pp in any scenario, then such a rule receives a negative rate in advance and further scenarios are not calculated at all. It helps to save significant amount of time.

Afterwards we tried to check which method of mutations, preventing from stucks in local extremes, caused better effects. In the base algorithm, if a mutation happens, a new gene value is drawn from a uniform distribution in a nearly random way. The only necessity

Table 1: Time of the computation of 100 scenarios in sec.

		maximum number of iterations				
		10^1	10^2	10^3	10^4	10^5
maximum possible	10^{-1}	0.94	1.19	1.24	1.22	1.21
relative difference	10^{-2}	1.12	7.73	22.74	28.74	28.15
between variables	10^{-3}	1.14	10.21	96.28	719.95	2070.89

Table 2: Share of years without a converged solution

		maximum number of iterations				
		10^1	10^2	10^3	10^4	10^5
maximum possible	10^{-1}	0.344	0.004	0.000	0.000	0.000
relative difference	10^{-2}	0.873	0.519	0.048	0.001	0.000
between variables	10^{-3}	0.910	0.904	0.836	0.476	0.044

concerns the parameters determining the NA thresholds - the first (warning) threshold should not take on higher value than the second (main) one. The alternative method assumes that a value of a mutated gene should slightly differ from an initial gene, i. e. it can increase or decrease by a unit. Thanks to this, new rules, even after a mutation, will not significantly differ from older rules. Thus a risk of a surge in assessments of new rules, due to random changes, is minimalised. Interpreting the results only from one course of the algorithm, it has to be said that the algorithm with 'intelligent' mutations does not bear better fruits than the original one. Figure 1 shows mean deviations of a structural balance from the MTO for the best rule in successive generations. The assessments of rules by the algorithm with 'intelligent' mutations do not converge faster than by the original one.

We also analysed how the results depend on macroeconomics assumptions, eg. formula of a function determining potential GDP. It can be expected intuitively that with a stable potential GDP growth, the results, in particular a structural balance to GDP ratio, should be more stable over time which should further imply better assessments of rules. The results presented in figure 1 confirmed those suppositions. Assuming a steady potential GDP growth, mean deviations from the MTO for the best rule were lower in every respective generation.

As it has been already written, every rule is assessed simultaneously based on 100 random scenarios - in order to check its behaviour both in good and bad times, depending on random formation of shocks affecting nominal and real variables. Scenarios are known in advance and they are the same for every rule in every generation. However, the assessment of the same rule under the same scenario differs between generations because GDP forecast errors are calculated each time in a random way. From this perspective, we investigate the results described in chapter 4. We calculated differences in assessments

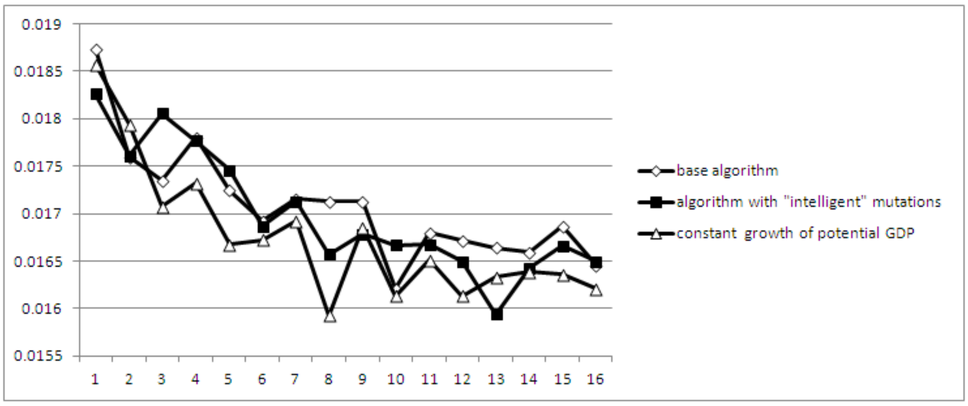

Figure 1: Comparison of a goal function value (vertical axis) of the best rule in successive generations (horizontal axis) between various algorithms and macroeconomic scenarios

of exactly the same rules but placed in different generations. The assessments differed but only to a little extent: the average assessment amounted to ca. 9%, while the standard deviation - 4%, whereas both the average difference and the standard deviation of differences amounted to as few as 0.6 pp. Anyway, multiple reassessing of the same rule prevented from a random assignment of a too positive assessment due to eg. extremely successful forecasts made in many scenarios concerning one generation.

In the initial variant it was assumed that an average assessment of 100 scenarios stands for an indication of rule's effectiveness. However, this assessing method can be called into question and replaced by a median or a truncated mean. Such an action would allow to eliminate an impact of extreme scenarios distorting rules' assessments. However, regardless of a speculation, whether including extreme scenarios are justified or not, they do not affect a final order of rules. In each of 20 generations, 100 rules were assessed based on 100 scenarios: by a mean, by a median, and by a 10%-truncated mean. Then, within every generation, Spearman rang correlation coefficients between 100 rules were computed. In every generation, both for a correlation between a mean and a median, as well as between a mean and a 10%-truncated-mean, very high (not lower than 97.6%) coefficients of a correlation were observed. That proves that a choice of a measure does not affect an order of rules according to their effectiveness.

Nevertheless, the most important decision made up thanks to the sensitivity analysis was a modification of the goal function. The results arising from a function minimising only deviations of a structural balance from the MTO turned out to be unacceptable in practice. First of all, they too often caused exceeding 3% GDP by a nominal deficit. It should not be allowed for exceedings to take place more than 10 times on average during 91-years, considered period, especially after application of a well-assessed rule. Caring only about deviations of a structural balance resulted in a rare correction of the NA, due to extremely high NA thresholds among others. Therefore, to the original goal function a constant component amounted to 0.02/number of scenarios was added for every exceeding of 3% GDP by a nominal deficit. Similarly, in case of exceeding this threshold, a square of the difference divided by the number of scenarios was also added to the goal function.

Owing to those penalties: 1) among well-assessed rules a nominal deficit exceeded 3% GDP very rarely; 2) the obtained parameter values were characterised by a more concentrated distribution, and 3) corrections of negative deviations from the NA happened more often thanks to the lower thresholds. The parameter values referring to the penalty for exceeding 3% GDP were set arbitrarily. Diminishing penalty parameters values would imply loosening of rule's restriction, i. e. it would imply a possibility of cumulating greater negative deviations or weaker cuts in expenditure dynamics. However, it would not change the general results stemming from the optimisation. As an example, reducing a weight of a penalty by reducing the first penalty component's value to 0.001 from 0.02 and multiplying the second penalty component's value by 0.01 instead of 1, increased the frequency of exceeding the 3%-GDP-threshold to ca. 4-10 from 2-3. The comment to the final results is placed in chapter 4.

The analysis presented above did not exhaust the set of questions connected with the optimisation of the algorithm. No alternative formulas for a potential GDP time series were investigated. We did not analyse the sensitivity of results on different average errors of GDP and inflation forecasts. In both cases another mechanism of generating forecasts could be implemented as well. Additionally, we did not analyse formation of shocks, in particular - their variances. We could also set a different composition of a new generation of organisms (rules), eg. eliminate inflow of new, totally random sets of the parameter values. Moreover, in the algorithm considered here, each gene can come from a first or a second 'parent' ('mother' or 'father') independently. Meanwhile, in genetic algorithms, the solution taking place also in the nature, so called 'crossing over' is used very often. During the 'crossing over', a set of new genes is build by sections deriving from 'mother' and 'father' alternately.

After the final formula of the algorithm was chosen, the results following from its application were interpreted, about which chapter 4 does treat.

4 Results of the applied genetic algorithm

4.1 Convergence of the results of the algorithm

The calculation gave rise to 69 generations of the rules. It follows from figures 2 an 3 that the algorithm achieved the convergence in the last quarter of this period. It can be found in figure 2, that not only are the goal function values getting lower from generation to generation, but the range of assessments is also stable after the initial convergence. Thus we can expect quite similar parameter values alongside with successive generations. At the same time, the parameter values should have been different enough in order not to get stuck in a local extreme. This expectation is also justified in particular by figure 5 which shows the history of values taken on by parameter L_{-}^{1}. The chart would anyway look similar for most other parameters. The histograms were built based on the best assessed 500 rules from those selected as 'parents'. The mean deviations of a structural balance from the MTO oscillate between 2.1% and 3.6% of GDP which constitute higher deviations by 0.5-1.5 pp than among the best rules obtained thanks to the goal function without penalising exceedings of 3% of GDP threshold. In return, the number of those exceedings is placed in general between 2.2 and 3.9 within 91 years taken into consideration. The following subchapters 6.2 - 6.5 describe the histograms of parameters of the best 500 rules obtained as the result of the application of the genetic algorithm. The best rule of all generations achieved the mean deviation of a structural balance from the MTO on the level of 2.6% of GDP and 2.4 exceedings of a 3%-of-GDP-threshold by a nominal deficit on average. It consisted mainly of the parameters' values being (or being close to) modes in their categories. This justifies to a certain extent why there is no analysis of interaction between various parameters in the further part of the chapter.

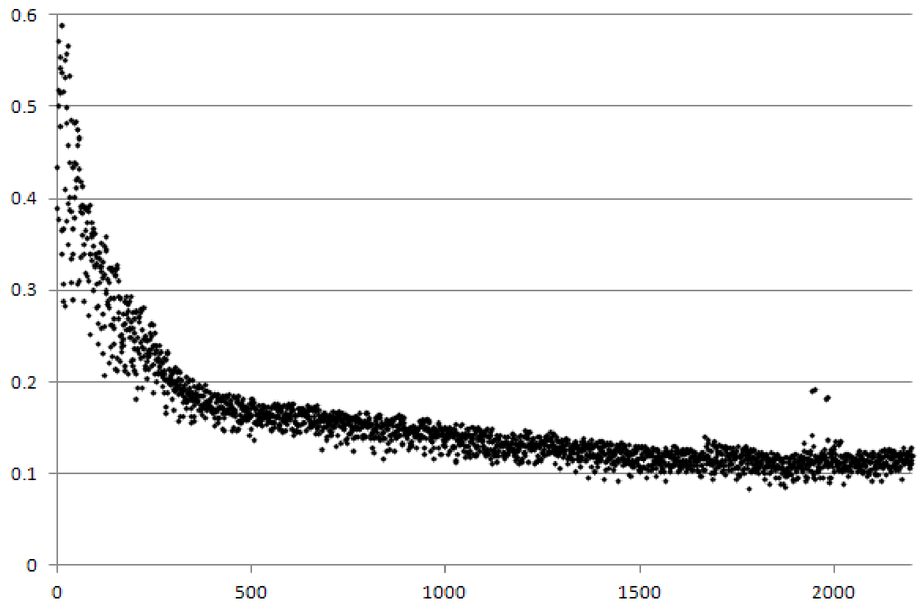

Figure 2: Goal function values (vertical axis) of the rules selected as 'parents' in the successive generations (horizontal axis)

4.2 Parameters p and f of the medium-term real GDP growth indicator

In case of parameters p and f, the optimisation results indicate that the medium-term real GDP growth indicator should be calculated as an average GDP dynamics of: 4 years preceding the year on which the expendiutre limit is imposed, the forementioned year and perhaps the forecast GDP dynamics of the one year after. In as many as 472 rules, out of the best 500, the parameter p took on 4, whereas in 235 rules parameter f amounted to 0 and in the remaining 265 rules f amounted to 1. It is difficult to explain exhaustively this strong repeatability. In order to verify the thesis that calculating the indicator every four years (due to the 4-year term of office in Poland) played here an important role, two versions of algorithms with slightly modified assumptions were applied. The first version assumed calculating the indicator every three years (the MTO is to be revised exactly so frequently). In this case, the parameter p predominantly took on 4 for the best rules but quite often, in 1/3 cases, it took on 5. In the second version, the indicator was calculated seperately for every year, so it reacted promptly on a change in a potential

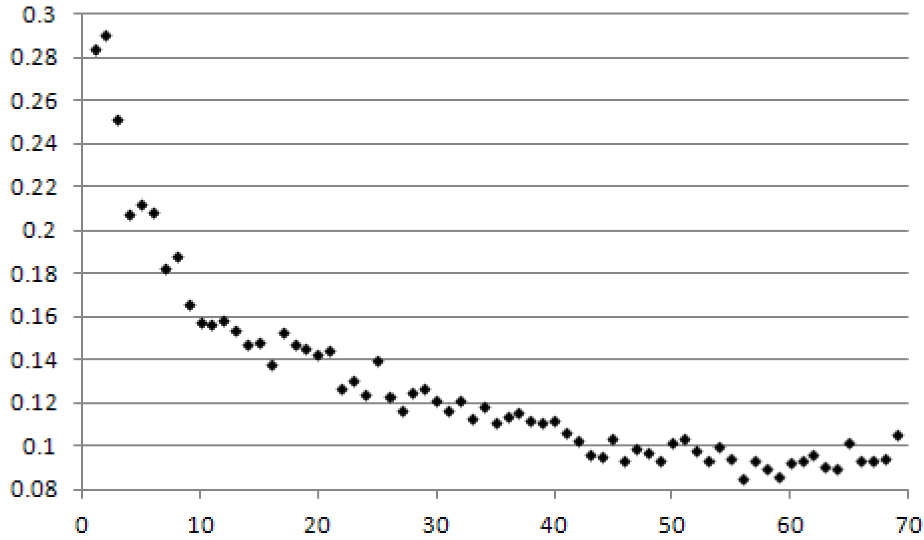

Figure 3: Goal function values (vertical axis) of the best rules in the succesive 70 generations (horizontal axis)

GDP growth. The most frequently observed values of the parameter p were: 3 or 4, and of f: 0 or 1. It is also worth to mention that thanks to such an indicator, the assessment of the best rule obtained in the whole course of the algorithm was significantly lower than of its counterpart obtained in the base algorithm. Ignoring future GDP dynamics can probably be caused by the assumed forecast errors distorting estimation of the potential GDP growth.

4.3 Notional account limits (thresholds)

In a long run a precise value of a notional account limit is nearly irrelevant. After all, there is no big difference, whether the NA totals 1% GDP, while a limit amounts to 0% or whether those numbers amount to respectively: -15% and -16%. Firstly, in both cases the risk of exceeding the threshold in future is the same. Secondly, the mean deviation of a structural balance from the MTO is calculated as a fraction. Its denominator increases alongside with years passing from a moment of an introduction of the rule, whereas a numerator is restricted, at least in the assumption, by the NA limit whose exceeding triggers off the correction mechanism. As a consequence, the mean deviation from the MTO converges toward zero independently from the threshold value.

Nevertheless, in a short run, determining threshold points for the NA is very important because it marks out space for cumulated deficits in which authorities can fit themselves without the necessity of cutting expenditures dynamics.

The obtained results for negative limits: L^1_- and L^2_- can be observed in figure 4 (upper row). The dominants for these parameters are respectively: 0% and -30% GDP. One could argue that those values were caused by getting stuck in a local extreme, which arose after a randomly generated, the best among first generations, rule possessed specific parameter values. Figure 5, which illustrates the history of taking on values by the parameters L^1_-

of the good rules (selected into 'parents' set) over generations, allows for refuting such arguements. The chart shows that over the long time the range, in which most values of L^1_- were situated, had been wide but then it systematically narrowed. In the first generations the values from the range 10 - 20% of GDP even outperformed others. Not before 40th generation, did the parameter values stabilise around 0% of GDP. However it cannot be ensured that a genetic algorith reaches a solution which is a global extreme.

With the assumed goal function, in which an exceeding of 3% of GDP threshold is penalised, the positive thresholds: L^1_+, L^2_+ contrary to the negative ones, took on values possibly high, i. e. close to 1 (see figure 4, bottom row). We conclude that surpluses on the NA should not be automatically allocated (removed). Otherwise, an allocation would induce a step growth of deficit. In such a way also the asymmetric rules in Switzerland and Germany look like - the mechanism corrects there only an excessive deficit.

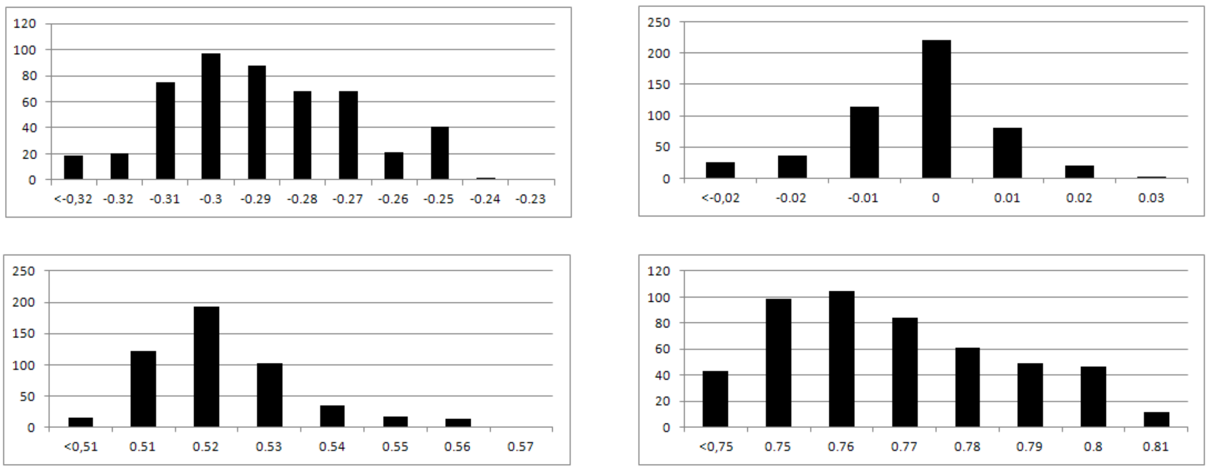

Figure 4: Histograms of parameters: $L^2_-, L^1_-, L^1_+, L^2_+$

4.4 Parameters α

The histograms concerning the parameters α - responsible for suspending a NA correction due to bad / good times, shown in figure 6, confirm conlusions from the previous subchapter. A correction of a negative cumulated deficit should happen often, while that of a positive one - seldom. According to the results of the algorithm, the optimal value for α^1_- would amount to 3.5 – 4.5 pp. Assuming the potential real GDP growth on the 4% level, the correction of expenditure dynamics, implied by the indicator, would be suspended only together with forecasted descrease in GDP or its stabilisation. This would mean a severe recession in Poland. Meanwhile, the histogram of parameter α^2_- is less conclusive. However, an impact of forecasts cannot be neglected here. Firstly, forecasts differ strongly (i. e. by 2-4 pp) from the medium-term GDP growth very seldom. An analysis of forecasts made by the European Commission between autumn 2003 and autumn 2011 can be treated here as an example. Among 26 values of the annual Polish GDP dynamics forecasted for two years ahead, only in two cases the forecasts differed from the 4% level more than by 1,5 pp. Both were forecasts of GDP dynamics for the year

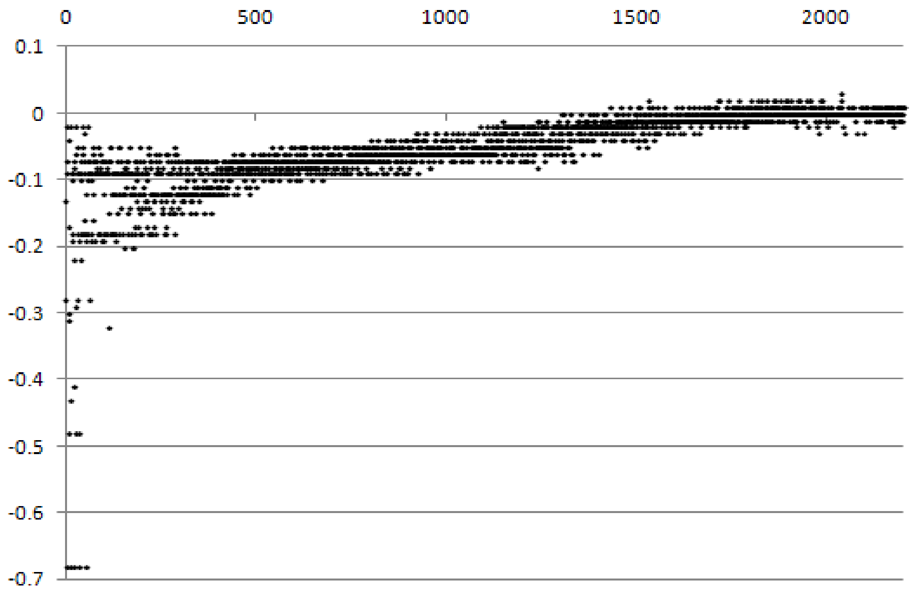

Figure 5: Values of the parameter L_{-}^{1} (vertical axis) of the rules selected in the successive generations (horizontal axis) to the 'parents' set

2010 made in the spring and autumn 2009 (0.8% and 1.8% respectively). Secondly, error forecasts quite often turn out to be large which bears risk of an improper decision (or its lack) about suspending a correction of expenditure dynamics. For example, in 2010 Polish GDP increased paradoxically by 3.8%. The outcomes for α_{+}^{1} and α_{+}^{2} are comparable with their counterparts from subchapter 4.3. According to those outcomes (see figure 6, bottom row) it would be enough if a forecasted dynamics of GDP exceeds the medium-term GDP growth by 0.5 - 2 pp, not to increase expenditure dynamics. Obviously, the values below zero should be thrown away from the set taken into consideration because they would (again paradoxically) force expenditure cuts in spite of a surplus collected on the NA.

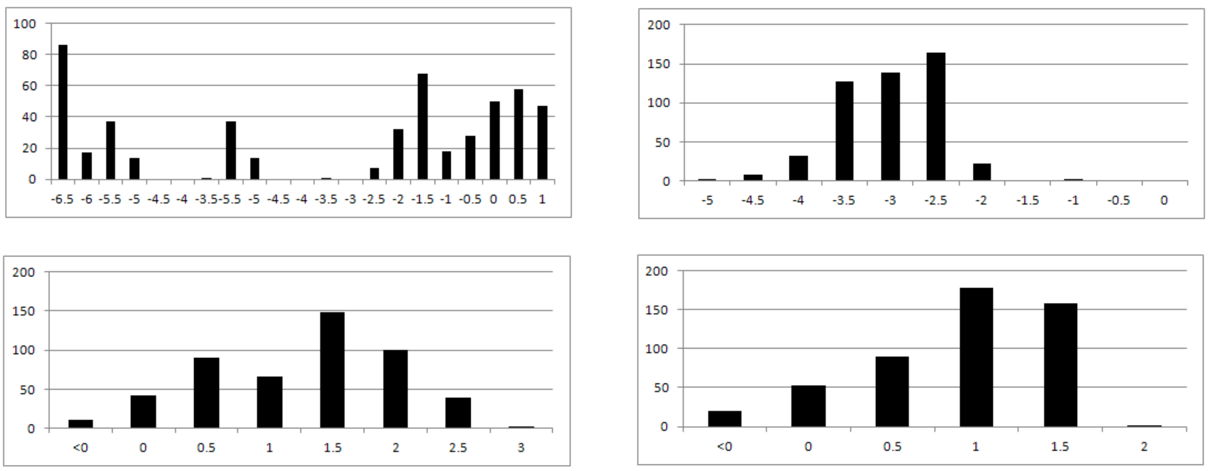

Figure 6: Histograms of parameters: $\alpha_{-}^{2}, \alpha_{-}^{1}, \alpha_{+}^{1}, \alpha_{+}^{2}$

4.5 Parameters β

The last group of the investigated parameters consist of the parameters β responsible for a size of a correction of a dynamics implied by the indicator, carried out after the NA limits are exceeded. The minor problem was to determine a size, expressed in percentage points, by which expenditure dynamics should be raised after exceeding the positive thresholds because these are very rare cases. The results indicate that both the limit β_+^1 and β_+^2 should not be positive. Exceeding treshold L_+^1 should even result in decreasing dynamics by 0.5 - 1 pp (442 cases among 500), while exceeding L_+^2 should imply no correction.

However, determining the parameters β_-^1 and β_-^2 seems to pose a much greater problem. This problem reflects the inherent trade-off in fiscal policy: stability of public finance vs. high economic growth. A certain suggestion regarding those parameters can be found in the revised Stability and Growth Pact (Regulation No 1175/2011). This legal act establishes that countries which have not achieved the MTO yet, are obliged to cut their expenditures in comparison with a potential GDP dynamics as much as it is necessary to reduce a structural balance by 0.5% of GDP annually. Assuming expenditures on the level of 50% of GDP, this reduction can be translated into 1 pp of an expenditures dynamics. According to the results of the algorithm, the reduction should be much deeper and amount to ca. 5.0 pp but the results are unambiguous for L_-^1. Two factors could give rise to such a sharp reduction. The goal function did not contain the component rewarding GDP growth or penalising for a recession, which is the first factor. Otherwise, too high structural deficit would be unacceptable as worsening, indirectly through an output gap, the goal function value. The second factor is an assumed lack of impact of fiscal policy on a potential (long-term) GDP growth. Perhaps the proper idea would be to make a size of a correction conditional on a forecasted GDP growth. It would allow to avoid procyclical fiscal policy, but meanwhile it would need to generate futher parameters α and further complication of the already complex rule.

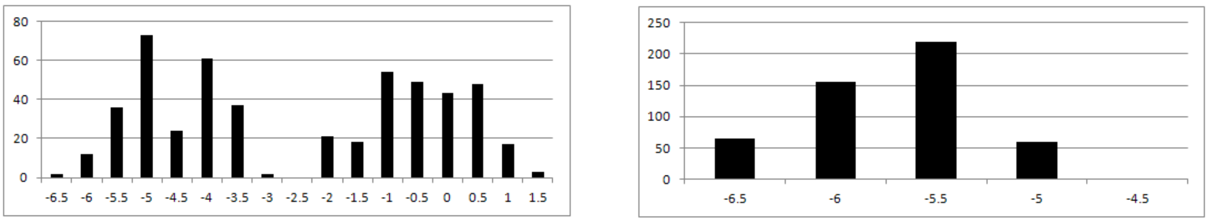

Figure 7: Histograms of parameters: β_-^2, β_-^1

5 Summary

The aim of this paper was to optimize the parameter values of the expenditure rule combined with the notional account (NA), introduction of which into the law was declared by the Polish government in several documents. The best rules, obtained thanks to the genetic algorithm, ensure a mean deviation of the structural balance from the MTO amounting to ca. 3%. They also lead to as few as 2-4 exceedings the 3% of GDP threshold point by a nominal deficit within considered, 91-year period since the introduction of the rule. Those rules treat a surplus accumulated on the notional account neutrally. Consequently, the positive NA limits are high, while a correction in case exceeding them - rather close to zero. On the other hand, a correction of accumulated deficits, should be much more strict. The most frequently observed values for the negative NA limits are -30% and 0% of GDP. After exceeding those limits, a correction of a GDP dynamics implied by the medium-term real GDP growth indicator, should amount to ca. 5 pp unless 'bad times' happen. The 'bad times' clause should be applied if a forecasted GDP dynamics is lower than a medium-term dynamics by 2.5 – 3.5 pp of GDP. The above-mentioned indicator, determining the level of an expenditure limit, should amount to an average annual GDP dynamics over 4 years before the year to which the limit refers and perhaps one dynamics after this year. The last recommendation is albeit weak, because it can result from the specific assumptions of the model.

Thus defined expenditure rule, with a mechanism correcting excessive cumulated deviations of a nominal balance from the MTO, would contribute to achieve stability of public finances in the long run.

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