Unconventional Monetary Policy  
and the Great Recession  
Estimating the Impact of a Compression in  
the Yield Spread at the Zero Lower Bound*  

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Abstract  
We explore the macroeconomic impact of a compression in the long-term bond yield spread within the context of the Great Recession of 2007-2009 via a Bayesian time-varying parameter structural VAR. We identify a ‘pure’ spread shock which, leaving the short-term rate unchanged by construction, allows us to characterise the macroeconomic impact of a compression in the yield spread induced by central banks’ asset purchases within an environment in which the short rate cannot move because it is constrained by the zero lower bound. Two main findings stand out.  

First, in all the countries we analyse (U.S., Euro area, Japan, and U.K.) a compression in the long-term yield spread exerts a powerful effect on both output growth and inflation.  
Second, conditional on available estimates of the impact of the FED’s and the Bank of England’s asset purchase programmes on long-term government bond yield spreads, our counterfactual simulations indicate that U.S. and U.K. unconventional monetary policy actions have averted significant risks both of deflation and of output collapses comparable to those that took place during the Great Depression.

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The decisive policy easing by the Fed and the ECB during the crisis, and the adoption of unconventional measures by the two central banks, was crucial in countering the threat of deflation in the current episode.

—Athanasios Orphanides

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1 Introduction

This paper tackles two questions:

- ‘How effective have central banks’ unconventional monetary policy actions been at countering the recessionary shocks associated with the 2007-2009 financial crisis?’

- ‘More generally, how powerful is monetary policy at the zero lower bound, once all traditional ammunition has been exhausted?’

We explore the macroeconomic impact of a compression in the long-term bond yield spread within the context of the Great Recession of 2007-2009 via Bayesian time-varying parameter structural VARs for the Euro area, the United States, Japan, and the United Kingdom. We identify a ‘pure’ spread shock which, leaving the short-term rate unchanged by construction, allows us to characterise the macroeconomic impact of a compression in long-term yield spreads induced by central banks’ asset purchase programmes within an environment in which the short rate cannot move because it is constrained by the zero lower bound.

Our main results may be summarised as follows.

First, in all the countries we analyse, a compression in the long-term yield spread exerts a powerful effect on both output growth and inflation.

Second, evidence clearly highlights the importance of allowing for time variation, as the impact of a spread compression exhibits, in several cases, important changes over the sample period. In the United States, for example, the impact on inflation exhibits three peaks corresponding to the Great Inflation of the 1970s, the recession of the early 1990s, and the most recent period, whereas the 1990s were characterised by a significantly weaker impact. By the same token, in the United Kingdom the impact on both inflation and output growth appears to have become stronger in recent years. This automatically implies that, for the present purposes, the use of fixed-coefficient models estimated over (say) the last two decades would offer a distorted picture, as it would under-estimate the impact resulting from yield spread compressions engineered

by central banks *via* asset purchase programmes in countering the recessionary shocks associated with the 2007-2009 financial crisis.

*Third*, conditional on Gagnon *et al.*’s (2010) estimates of the impact of the FED’s asset purchase programme on the 10-year government bond yield spread,\(^2\) our counterfactual simulations indicate that U.S. unconventional monetary policy actions have averted significant risks both of deflation and of output collapses comparable to those that took place during the Great Depression. The same holds for the United Kingdom conditional on Charlie Bean’s (2009) broad estimate of the impact of the Bank of England’s asset purchase programmes on long-term yield spreads.\(^3\)

### 1.1 Related literature

The results of this paper can be linked to some recent contributions in the literature. Evidence, albeit reduced form, for a negative relationship between the term premium and real economic activity is provided by Rudebusch, Sack, and Swanson (2007) who show that a decline in the term premium of 10-year Treasury yields tends to boost GDP growth. Gilchrist, Yankov, and Zakrajsek (2009) study the transmission of credit spread shocks originating in the corporate bond market to the broader economy within a structural framework. Based on a factor-augmented VAR model, they demonstrate that an unexpected widening of credit spreads leads to a significant contraction of economic activity and a fall in prices. Extending their model to allow the interaction between the financial sector and the macroeconomy to evolve over time, Amir-Ahmadi (2009) presents evidence of substantial changes in the responses of key macroeconomic variables to credit spread shocks, the strength of which appear to be associated with periods of higher and lower financial market volatility and the state of the business cycle. However, since both studies allow identified spread shocks to trigger a reaction in the policy rate, no consideration is given to the possibility of monetary policy being constrained by the zero lower bound.

A recent empirical contribution that is closest in spirit to ours in that it investigates the macroeconomic effect of a decline in interest rate spreads for a *given* level of the policy rate is Lenza, Pill, and Reichlin (2010). They focus on the likely impact of the ECB’s unconventional policy actions on the short-end of the yield curve by contrasting macroeconomic outcomes resulting from a policy versus a no-policy scenario which differ only in the evolution of short-term interest rates. Put differently, in order to gauge the effectiveness of policy intervention in warding off disastrous macroeconomic consequences, they conduct a forecasting exercise of key variables based on a reduced-form VAR model conditional upon a counterfactual and an observed path of money market rates. However, their analysis builds on the premise that the underlying behavioural relationships have not been affected by the crisis, which stands

\(^2\)See Gagnon, Raskin, Remache, and Sack (2010).

\(^3\)See Bean (2009).
in stark contrast to our findings of remarkable time variation over the whole sample period, not just during the most recent episode of financial turmoil. While they show that the narrowing of spreads induces economic stimulus, these beneficial effects take hold only after a considerable delay.

DelNegro, Eggertsson, Ferrero, and Kiyotaki (2010) propose to quantify the effect of non-standard policy measures in a general equilibrium model that features credit frictions with an explicit role for the zero bound on nominal interest rates. In particular, they show (by means of simulations calibrated on the actual size of liquidity provisions and purchase programmes) that changes in the relative supply of liquid and illiquid assets available in the economy, as a result of unconventional policy operations, helped avoid a second Great Depression by counteracting deflationary tendencies and the drop in output. They observe that the effects of policy-induced liquidity shocks are more powerful when the zero lower bound is binding, thereby emphasizing the need to take this constraint into account when evaluating the impact of a compression in the yield spread.

The remainder of the paper is organised as follows. The next section describes the key features of the reduced-form VAR model we are using herein, it discusses the reasons behind such a modelling choice, and it illustrates both the identification scheme and the motivation behind it. Section 3 presents the empirical evidence. Section 4 concludes.

2 Methodology

2.1 A Bayesian time-varying parameter VAR with stochastic volatility

In what follows, we will work with the time-varying parameter VAR(p) model used by Benati (2010b), which is a slightly modified version of the one used by Cogley, Primiceri, and Sargent (2010),

\[
Y_t = B_{0,t} + B_{1,t}Y_{t-1} + \ldots + B_{p,t}Y_{t-p} + \epsilon_t \equiv \mathbf{X}_t^\prime \theta_t + \epsilon_t
\]

where the notation is obvious, with \(Y_t \equiv [r_t, s_t, \pi_t, y_t]'\), with \(r_t\), \(s_t\), \(\pi_t\), and \(y_t\) being the short-term (policy) rate, the 10-year government bond yield spread, GDP deflator inflation, and real GDP growth, respectively. (For a description of the data, see Appendix A.)

Consistent with the vast majority of papers in the literature, and mostly for reasons of computational feasibility, the lag order is set to \(p=2\). Following, e.g. Cogley and Sargent (2002), Cogley and Sargent (2005), and Primiceri (2005) the VAR’s time-varying parameters, collected in the vector \(\theta_t\), are postulated to evolve according to

\[
p(\theta_t \mid \theta_{t-1}, Q_t) = I(\theta_t) f(\theta_t \mid \theta_{t-1}, Q_t)
\]
with $I(\theta_t)$ being an indicator function rejecting unstable draws—thus enforcing a stationarity constraint on the VAR\(^4\)—and with $f(\theta_t \mid \theta_{t-1}, Q_t)$ given by

$$\theta_t = \theta_{t-1} + \eta_t$$

with $\eta_t = [\eta_{1,t}, \eta_{2,t}, \ldots, \eta_{N(1+Np),t}]'$ where $\eta_t \sim N(0, Q_t)$. We postulate a stochastic volatility specification for the evolution of the covariance matrix of the innovations to the VAR’s random-walk coefficients, $Q_t$. Specifically, we assume that $Q_t$ is given by

$$Q_t = \begin{bmatrix} q_{1,t} & 0 & \cdots & 0 \\ 0 & q_{2,t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{N(1+Np),t} \end{bmatrix}$$

with the $q_{i,t}$’s evolving as geometric random walks: $\ln q_{i,t} = \ln q_{i,t-1} + \omega_{i,t}$. This specification is simpler than the one used by Cogley, Primiceri, and Sargent (2010), who factor the covariance matrix of the innovations to the VAR’s random-walk parameters as $Q_t = (B_s^{-1})'H_{s,t}B_s^{-1}$, where $H_{s,t}$ has exactly the same specification which is postulated herein for $Q_t$, and $B_s$ is a triangular matrix with ones along the main diagonal and static covariance parameters below. (To put it differently, our specification is obtained from Cogley et al.’s by setting $B_s$ equal to the identity matrix.) A key reason for simplifying Cogley et al.’s model along this dimension is that, as we discuss shortly, we are significantly complicating it along a crucial dimension—that is, we are allowing for time variation in the off-diagonal elements of the VAR’s covariance matrix of reduced-form innovations as in Primiceri (2005). For future reference, we define $q_t = [q_{1,t}, q_{2,t}, \ldots, q_{N(1+Np),t}]'$.

The VAR’s reduced-form innovations in (1) are postulated to be zero-mean normally distributed, with time-varying covariance matrix $\text{Var}(\varepsilon_t) \equiv \Omega_t$ which, following established practice, we factor as

$$\Omega_t = A_t^{-1}H_t(A_t^{-1})'$$

The time-varying matrices $H_t$ and $A_t$ are defined as:

$$H_t = \begin{bmatrix} h_{1,t} & 0 & 0 & 0 \\ 0 & h_{2,t} & 0 & 0 \\ 0 & 0 & h_{3,t} & 0 \\ 0 & 0 & 0 & h_{4,t} \end{bmatrix}, \quad A_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21,t} & 1 & 0 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 & 0 \\ \alpha_{41,t} & \alpha_{42,t} & \alpha_{43,t} & 1 \end{bmatrix}$$

with the $h_{i,t}$ evolving as geometric random walks,

$$\ln h_{i,t} = \ln h_{i,t-1} + \nu_{i,t}$$

\(^4\)It is important to be precise here about the meaning of such a stationarity constraint. Although, due to the time-varying parameter specification (1), inflation contains a stochastic trend, the constraint (2) implies that its fluctuations around such trend cannot be explosive.
For future reference, we define $h_t \equiv [h_{1,t}, h_{2,t}, h_{3,t}, h_{4,t}]'$. Following Primiceri (2005), we postulate the non-zero and non-one elements of the matrix $A_t$—which we collect in the vector $\alpha_t \equiv [\alpha_{21,t}, ..., \alpha_{43,t}]'$—to evolve as driftless random walks,

$$\alpha_t = \alpha_{t-1} + \tau_t,$$

and we assume the vector $[u_t, \tau_t, \nu_t, \omega_t]'$ to be distributed as $N(0, V)$, with

$$V = \begin{bmatrix} I_4 & 0 & 0 & 0 \\ 0 & S & 0 & 0 \\ 0 & 0 & Z_\nu & 0 \\ 0 & 0 & 0 & Z_\omega \end{bmatrix}, \quad Z_\nu = \begin{bmatrix} \sigma_{\nu,1}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\nu,2}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\nu,3}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\nu,4}^2 \end{bmatrix}$$

and

$$Z_\omega = \begin{bmatrix} \sigma_{\omega,1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{\omega,2}^2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sigma_{\omega,N-(1+Np)}^2 \end{bmatrix}$$

where $u_t$ is such that $\epsilon_t \equiv A_t^{-1}H_t^{1/2}u_t$. Finally, following Primiceri (2005) we adopt the additional simplifying assumption of postulating a block-diagonal structure for $S$, too—namely

$$S \equiv \text{Var}(\tau_t) = \begin{bmatrix} S_1 & 0_{1 \times 2} & 0_{1 \times 3} \\ 0_{2 \times 1} & S_2 & 0_{1 \times 3} \\ 0_{3 \times 1} & 0_{3 \times 2} & S_3 \end{bmatrix}$$

with $S_1 \equiv \text{Var}(\tau_{21,t})$, $S_2 \equiv \text{Var}([\tau_{31,t}, \tau_{32,t}'])$, and $S_3 \equiv \text{Var}([\tau_{41,t}, \tau_{42,t}, \tau_{43,t}]')$, thus implying that the non-zero and non-one elements of $A_t$ belonging to different rows evolve independently. As discussed in Primiceri (2005, Appendix A.2), this assumption drastically simplifies inference, as it allows to do Gibbs sampling on the non-zero and non-one elements of $A_t$ equation by equation.

### 2.1.1 Rationale for using the proposed reduced-form VAR

As we just mentioned, the time-varying parameter VAR we use herein features a stochastic volatility specification for both the VAR’s reduced-form innovations, and the innovations to the VAR’s random-walk parameters.

The key reason for using a time-varying parameter specification is to be able to recover impulse-response functions (henceforth, IRFs) which are localised in time, thus allowing us to characterise the impact of a compression of the bond yield spread induced by central bank’s unconventional monetary policies during the Great Recession of 2007-2009. In this respect, the use of fixed-coefficient models would be especially disadvantageous both (i) at a very general level, in light of the widespread evidence of instability in macroeconomic time series;\(^5\) and (ii) specifically within the present

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context, because the notion that the dramatic economic contraction associated with the Great Recession has left key structural macroeconomic relationships unchanged is entirely open to question. To put it differently, postulating that structural economic dynamics have remained unchanged in the face of such a severe macroeconomic dislocation—which is what is implicitly done when using fixed-coefficient models—is essentially a leap of faith.⁶

Having provided the rationale for using a time-varying parameter specification for the reduced-form VAR, we now ought to discuss the need for a specification with a time-varying extent of drift, which is what a stochastic-volatility specification for the innovations to the random-walk parameters delivers. The reason here is straightforward: as a simple inspection of the raw macroeconomic data reveals, key macroeconomic variables—first and foremost, interest rates, inflation, and output growth—have been remarkably volatile during the Great Inflation of the 1970s, extremely stable during the Great Moderation period, and, in the case of output growth and interest rates, once again very volatile during the Great Recession. The traditional, ‘first-generation’ time-varying parameter models—see in particular, Cogley and Sargent (2002), Cogley and Sargent (2005), and Primiceri (2005)—have a hard time fitting such a pattern of time variation successfully, as they postulate that the extent of random-walk drift is constant along the sample. As a result, they tend to ‘under-drift’ (that is, to drift too little) during the Great Inflation period, and to ‘over-drift’ (that is, to drift too much) during the Great Moderation period, thus automatically distorting inference. (Based on our own experience, this is especially apparent for the Euro area: evidence on this is available upon request.) The logical solution is a model with a time-varying extent of drift, which, thanks to its flexibility, is capable of capturing changes over time in the macroeconomic structure.

2.2 Estimation

We estimate (1)-(10) via standard Bayesian methods. Appendix B discusses our choices for the priors, and the Markov-Chain Monte Carlo algorithm we use to simulate the posterior distribution of the hyperparameters and the states conditional on the data.

⁶In this respect, the fact that the model used herein features a time-specific extent of random-walk time variation in the VAR’s coefficients is especially important, as it allows the dynamics of the VAR’s coefficients ‘to lay dormant’ for comparatively long periods—so that during those quarters the model approximates a fixed-coefficient VAR—and then to pick up speed in a data-driven way, as the information contained in the sample suggests.
2.3 Assessing the convergence of the Markov chain to the ergodic distribution

Following Primiceri (2005), we assess the convergence of the Markov chain by inspecting the autocorrelation properties of the ergodic distribution’s draws. Specifically, in what follows, we consider the draws’ inefficiency factors (henceforth, IFs), defined as the inverse of the relative numerical efficiency measure of Geweke (1992),

\[
RNE = (2\pi)^{-1} \frac{1}{S(0)} \int_{-\pi}^{\pi} S(\omega) d\omega
\]

where \( S(\omega) \) is the spectral density of the sequence of draws from the Gibbs sampler for the quantity of interest at the frequency \( \omega \). We estimate the spectral densities by smoothing the periodograms in the frequency domain by means of a Bartlett spectral window. Following Berkowitz and Diebold (1998), we select the bandwidth parameter automatically via the procedure introduced by Beltrao and Bloomfield (1987).

Figures 1-4 show, for the Euro area, the United States, and Japan, based on the 10-year government bond yield spread, and for the United Kingdom, based on the ‘long-term government bond yield’ spread, the draws’ IFs for the models’ hyperparameters—i.e. the free elements of the matrices \( Z_0, Z_\omega, S_1, S_2, \) and \( S_3 \)—and for the states, i.e. the time-varying coefficients of the VAR (the \( \theta_i \)'s), the volatilities of the innovations to the VAR’s random-walk parameters (the \( q_{i,t} \)'s), the volatilities of the VAR’s reduced-form innovations (the \( h_{i,t} \)'s), and the non-zero and non-one elements of the matrix \( A_t \). As the figures show, for all countries the autocorrelation of the draws is uniformly very low, being in the vast majority of cases around or below 3,\(^7\) thus suggesting that the Markov chains have indeed converged.

2.4 How reasonable are our priors? An informal assessment

Since the reliability of our results ultimately depends upon the meaningfulness of the assumptions underlying our analysis, it is important to get an idea about how reasonable our priors in fact are (the next sub-section, on the other hand, discusses our identification assumptions). A simple and informal check we routinely use when we work with time-varying parameter VARs, in order to assess the reasonableness of the Bayesian priors, is to look at the inflation trends produced by the model. For the United States, in particular, there is a vast consensus—stemming, first and foremost, from the work of Cogley and Sargent—that trend inflation peaked, during the 1970s, between 7 and 8 percent, and significantly declined since then. If the Bayesian priors underlying our analysis are in any way reliable, the estimated time-varying VAR for the United States should generate comparable results.

\(^7\)As stressed by Primiceri (2005, Appendix B), values of the IFs below or around twenty are generally regarded as satisfactory.
Figure 5 plots, for the Euro area, the United States, Japan, and the United Kingdom, actual GDP deflator inflation together with the time-varying trends generated by the model. Several things are apparent from the figure. In particular, first, concerning the United States, the median inflation trend peaks at about 7 percent during the second half of the 1970s, exactly in line with the just-mentioned previous evidence. Second, in general, the estimated inflation trends manifestly appear to capture the slow-moving, low-frequency component of inflation, and are indeed very strongly correlated with simple heuristic measures of trend inflation such as the Hodrick-Prescott trend. In particular, in Japan, where the GDP deflator has decreased, as of 2009Q4, by 14.3 percent compared with the peak reached in 1994Q2, the period of deflation following the mid-1990s is especially apparent, with trend inflation estimated, at the end of 2009, at minus 1.5 percent.

2.5 Identification

We achieve identification by imposing a mixture of sign restrictions and zero restrictions on impact. Specifically, we identify three ‘traditional’ shocks—monetary policy, demand non-policy, and supply—via a standard set of sign restrictions (see e.g. Benati (2008) and Benati and Goodhart (2010))—together with an additional shock to the spread which, by construction, is postulated to leave the policy rate unchanged, and is therefore recovered via a zero restriction on impact. A key point to stress is that, for the present purposes, the identification of a ‘pure’ spread shock—which, by construction, leaves the short-term rate unchanged—is of crucial importance, as it allows us to explore the impact of a compression of the yield spread within an environment in which the policy rate is bound to stay unchanged for an extended period (in what follows, we will leave it unchanged for 8 quarters after the impact). Both the sign restrictions and the zero restriction are imposed only on impact.

The set of restrictions on the structural impact matrix at zero is summarised in the following table. It can be trivially shown that this set of restrictions is sufficient to separate the various shocks from one another, thus achieving identification.

<table>
<thead>
<tr>
<th>Shock:</th>
<th>Variable:</th>
<th>$\epsilon^M_i$</th>
<th>$\epsilon^{SP}_i$</th>
<th>$\epsilon^D_i$</th>
<th>$\epsilon^S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short rate</td>
<td>$\geq 0$</td>
<td>0</td>
<td>$&gt; 0$</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Spread</td>
<td>$\leq 0$</td>
<td>$&gt; 0$</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Inflation</td>
<td>$\leq 0$</td>
<td>$&lt; 0$</td>
<td>$\geq 0$</td>
<td>$\leq 0$</td>
<td></td>
</tr>
<tr>
<td>Output growth</td>
<td>$\leq 0$</td>
<td>$&lt; 0$</td>
<td>$\geq 0$</td>
<td>$\geq 0$</td>
<td></td>
</tr>
</tbody>
</table>

8 See e.g. Canova and DeNicolo (2002), Faust (1998), Peersman (2005), and Uhlig (2005).
9 This implies that agents expect the zero bound to be binding for a period of 8 quarters. According to DelNegro, Eggertsson, Ferrero, and Kiyotaki (2010), this is a reasonable assumption for the duration of the constraint as it is in line with survey evidence of market participants during the crisis.
We compute the time-varying structural impact matrix, $A_{0,t}$, by combining the procedure proposed by Rubio-Ramirez, Waggoner, and Zha (2010) for imposing sign restrictions with the imposition of a single zero restriction via a deterministic rotation matrix. Specifically, let $\Omega_t = P_t D_t P_t'$ be the eigenvalue-eigenvector decomposition of the VAR’s time-varying covariance matrix $\Omega_t$, and let $\tilde{A}_{0,t} = P_t D_t^{1/2}$. We draw an $N \times N$ matrix, $K$, from the $N(0, 1)$ distribution, we take the QR decomposition of $K$—that is, we compute matrices $Q$ and $R$ such that $K = QR$—and we compute the time-varying structural impact matrix as $\tilde{A}_{0,t} = \tilde{A}_{0,t} \cdot Q$. We then impose a zero in the $(1,2)$ position of $\tilde{A}_{0,t}$ via an appropriate rotation of $\tilde{A}_{0,t}$. Specifically, by defining a rotation matrix $\tilde{R}$ as

$$\tilde{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0_2 \\ \sin(\theta) & \cos(\theta) & 0_2 \\ 0_2 & I_2 & \end{bmatrix}$$ (12)

with $\tilde{R} \cdot \tilde{R}' = I_4$ (with $I_4$ being the $4 \times 4$ identity matrix), where $\theta = \tan^{-1}(\tilde{A}_{0,t}^{1,2}/\tilde{A}_{0,t}^{1,1})$, where $\tilde{A}_{0,t}^{i,j}$ is the $(i,j)$ element of $\tilde{A}_{0,t}$, we have that $A_{0,t} = \tilde{A}_{0,t} \cdot \tilde{R}$ has a zero in the $(1,2)$ position. If $A_{0,t}$ satisfies the sign restrictions—which, by construction, were satisfied by $\tilde{A}_{0,t}$—we keep it, otherwise we discard it and we repeat the procedure until we obtain an impact matrix which satisfies both the sign restrictions and the zero restriction at the same time.

### 2.5.1 Rationale for the identification scheme

As we just mentioned, the three ‘traditional’ shocks are identified via a standard set of sign restrictions. As it is well known, especially from the work of Fabio Canova and his co-authors, a key advantage of sign restrictions compared to alternative identification schemes based on restrictions on impact—for example, Cholesky for identifying monetary policy shocks—is that they are, in principle, fully compatible with general equilibrium (that is, DSGE) models, whereas for alternative identification schemes this is not necessarily the case. Canova and Pina (2005), in particular, provide dramatic illustrations of how applying Cholesky, in order to identify monetary policy shocks, to series generated by a standard DSGE model dramatically distorts inference, for example generating ‘price puzzles’ which are not in the original data-generation process. Although, in principle, demand and supply shocks could be recovered via alternative schemes, our need to be able to identify, on top of them, the monetary policy shock naturally leads us towards sign restrictions. On the other hand, the very nature of the question we are trying to answer—‘What is the macroeconomic impact of a spread compression in a situation in which the central bank leaves the policy rate unchanged?’—logically implies that the spread shock, which is

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10 See at http://home.earthlink.net/~tzha02/ProgramCode/SRestrictRWZalg.m.
11 See e.g. Canova and Pina (2005) and Canova (2007).
12 See e.g. Blanchard and Quah (1989).
the key object of interest here, cannot possibly be recovered via sign restrictions, and can only be extracted from the data by means of a zero restriction on impact.

3 Evidence on the Impact of a Compression in the Yield Spread

In this section, we tackle two groups of questions. First, conditional on available estimates of the impact of central banks’ asset purchase programmes on long-term government bond yield spreads, what role did unconventional monetary policy play within the context of the 2007-2009 Great Recession? In particular, did central banks’ unconventional monetary policy actions avert significant risks of deflation and of output contractions, on a scale comparable to those which took place during the Great Depression?

Second, how large is the impact of a compression in the long-term yield spread on inflation and output growth within an environment in which the short-term (policy) rate does not move, because—in the present context—it is constrained by the zero lower bound? And has such impact changed over time?

We address the two groups of issues in turn.

3.1 Did unconventional monetary policies avert catastrophic outcomes?

In tackling the issue of whether central banks’ unconventional monetary policy actions have averted significant risks of deflation and large-scale output contractions we uniquely focus on the U.S. and U.K. experience. There are two reasons for that. First, in both countries central banks have been very explicit about their goal of ‘flattening the yield curve’ (that is, compressing the long-short spread) via asset purchases in order to stimulate aggregate demand. That was not the case, for example, for the Euro area, and for the ECB’s ‘enhanced credit support’ policy, which never had, as its fundamental objective, to compress spreads in order to jolt aggregate demand. Rather, the key objective of the ECB’s policy has always been to provide proper support to dysfunctional credit markets, so that a decrease in the spreads would only ultimately come as a result of a normalisation of the situation. Second, and crucially, for the United States and the United Kingdom we can rely on Gagnon et al.’s and Bean’s estimates of the impact of quantitative easing policies on the term spread.

3.1.1 The United States

In their extensive empirical analysis of the impact of the FED’s asset purchase programmes on U.S. long-term yield spreads, Gagnon, Raskin, Remache, and Sack (2010) conclude that
‘[...] these purchases caused economically meaningful and long-lasting reductions in longer-term interest rates on a range of securities, including on securities that were not included in the purchase programs. [...] Our results [based on time-series methods] suggest that the $1.725 trillion in announced purchases reduced the 10-year term premium by between 38 and 82 basis points. This range of point forecasts overlaps considerably with that obtained in our event study, which is impressive given that entirely separate data and methodologies were used to obtain the results.’

In what follows, we take Gagnon et al.’s (2010) time-series estimates as our benchmark measure of the impact of the FED’s asset purchase programmes on U.S. long-term yield spreads—specifically, for illustrative purposes we will consider the average between their lower and upper estimates of the impact on the 10-year government bond yield spread, that is 60 basis points—and we will tackle the following questions:

‘What would have happened if the FED had not engineered such a yield spread compression via asset purchases? Specifically, would the U.S. economy have fallen into deflation? Would the output collapse have been comparable to the one that took place during the Great Depression?’

Figure 6 reports results from the following counterfactual simulation. Starting in 2009Q1, we re-run history
(i) conditional on the time-varying VAR’s estimated coefficients,
(ii) keeping all the structural shocks except the one to the spread unchanged at their estimated historical values, and
(iii) rescaling the shocks to the spread in such a way that the counterfactual path for the spread is, for the whole of 2009, 60 basis points higher than the actual historical path.

An important point to stress about this counterfactual simulation is that, since we are only manipulating structural shocks, while leaving all other elements of the estimated SVAR unchanged—including, first and foremost, the monetary policy rule—such a counterfactual is not vulnerable to Sargent’s (1979) criticism of SVAR-based policy counterfactuals (for a discussion, see Section 3.2.1).

The first and last panels of the figure report actual inflation and real GDP growth, together with the medians and the one-standard-deviation percentiles of the distributions of counterfactual inflation and real GDP growth, respectively, whereas the middle panel shows the fractions of draws from the posterior distribution for which the economy is in deflation (it is worth stressing that, during 2009, actual GDP deflator inflation never went negative). The figure portrays a sobering picture of what might have been had the FED not engineered yield spread compressions via its asset purchase programmes. Specifically, based on median estimates, macroeconomic performance would have clearly been worse, with inflation slipping slightly below zero, and output growth reaching a trough of almost minus 10 percent in the first quarter
of 2009. What is especially noteworthy of Figure 6, however, are not the median projections, but rather the risks associated with such projections—that is, the profiles of the entire distributions. Concerning inflation, in particular, the fraction of draws for which counterfactual inflation would have been negative peaks at about 90 per cent in 2009Q2, and stays consistently beyond 65 percent over the next two quarters. As for output growth, results are even more ominous, with the one-standard-deviation lower percentile reaching minus 17 percent. Although these figures may appear, at first blush, wildly implausible, it is important to keep in mind that in the fourth quarter of 1929 U.S. real GNP contracted, on a quarter-on-quarter annualised basis (that is, the measure we are using here), by a remarkable 17.5 percent, whereas over the three subsequent years (from 1930Q1 to 1932Q4) the average quarter-on-quarter annualised rate of growth was equal to minus 10.4 percent. So, although the results portrayed in Figure 6 are outside the bounds of advanced countries’ post-WWII experience, they are definitely not outside the bounds of historical experience, and on the contrary, they are exactly in line with the experience of the U.S. Great Depression.

Rather, it is even possible to make a strong case that such counterfactuals are, along one specific dimension, excessively optimistic, in the following sense. When performing counterfactuals—either with SVARs, or with DSGE models—an important implicit assumption behind the entire exercise is that the estimated structural shocks are truly structural, in particular in the sense of being invariant to changes in policy. Although this assumption is a plausible one under normal circumstances, within the present context it might legitimately be regarded as questionable. Consider, in particular, the case of demand non-policy shocks. The estimated sequence of these shocks for 2009 is conditional on the FED having announced and implemented its asset purchase programmes, which, among other things, contributed to ‘calm nerves’ and to steady markets. Suppose, however, that the FED had stood idle in the face of the crisis: is it reasonable to assume that, under these circumstances, business and consumer confidence would have been the same as they have historically been? Such an assumption is, in our view, a pretty heroic one, which automatically implies that the resulting collapse in confidence would most likely have led to an alternative—and ‘worse’—sequence of demand non-policy shocks, and therefore, as a consequence, to worse macroeconomic performance across the board. We therefore conjecture that, rather than being unrealistically dire, our counterfactual scenario might in fact be too rosy, and that things might have turned worse.

3.1.2 The United Kingdom

In a speech delivered in May 2009, the Bank of England’s Deputy Governor, Charlie Bean, thus spoke of the impact of the Bank’s asset purchase programme on long-term

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13These figures are based on the real GNP data found in Balke and Gordon (1986), appendix B, Table 2.
There are signs that these measures are having a beneficial impact [...] Spreads on commercial paper eligible for purchase have fallen by around $\frac{1}{2}$ percentage point and the size of the market has increased by around 10%. Similarly, average spreads on sterling investment grade corporate bonds for industrial companies have declined by some 60 basis points and gross issuance of bonds by UK companies has been strong. These developments may reflect a range of influences, but feedback from market participants suggests that our purchases have indeed played a helpful role.

His assessment is supported by empirical evidence presented in Meier (2009) who purports that the Bank’s purchases of UK government bonds have reduced gilt yields by a range of at least 35-60 basis points.

Figure 7 reports, for the United Kingdom, results from the same exercise we discussed in the previous paragraph for the United States, in which we re-run the U.K. Great Recession based on the estimated SVAR, rescaling the spread shocks for 2009 in such a way that the counterfactual path for the spread is 50 basis points higher than it has historically been. Unsurprisingly, results are in line with those for the U.S., and in fact, they are even more ominous, with much stronger deflation, and a significantly deeper recession, reaching, in the first quarter of 2009, about minus 19 percent. Once again, it is possible to make a convincing argument that these projections are actually optimistic, exactly for the same reason we previously highlighted for the U.S.

3.2 How powerful is a compression in the yield spread at the zero lower bound?

3.2.1 Results obtained by ‘zeroing out’ the structural VAR’s monetary rule

Figures 8, 10, 12, and 14 show, for the U.S., the Euro area, Japan, and the U.K., respectively, the median time-varying impulse-response functions (henceforth, IRFs) of the yield spread, GDP deflator inflation, and real GDP growth, to a one-percent negative shock to the spread for all available quarters. Figures 9, 11, 13, and 15, on the other hand, show, for the same countries and variables, and for selected quarters, the median IRFs to a one-percent negative shock to the spread, together with the 16th and 84th percentiles. Within the present exercise the short-term (policy) rate has been kept at zero both on impact (by construction), and for the subsequent eight quarters by ‘zeroing out’ the structural VAR’s monetary rule as follows.

In the exercise we are performing, the long-term yield spread is subject to a one-time shock equal to minus one percent, whereas the short-term rate remains

\[ \text{14See Bean (2009).} \]
unchanged both on impact—which we implement by construction, by the very way we extract such ‘pure’ spread shocks—and over the subsequent eight quarters. We implement the restriction that the short-term rate stays unchanged for eight quarters after the impact by setting to zero all the coefficients in the structural VAR’s monetary rule, with the single exception of the one on the short rate. To fix ideas, let the structural VAR (henceforth, SVAR) representation be given by

\[ A_0^{-1}Y_t = A_0^{-1}B_1Y_{t-1} + \ldots + A_0^{-1}B_pY_{t-p} + \epsilon_t \]  

where \( Y_t = [R_t, X_t']' \) is an \( N \times 1 \) vector of endogenous variables, with \( R_t \) being the nominal short-term rate and \( X_t \) being an \( (N-1) \times 1 \) vector of variables other than \( R_t \), including, in the present case, the spread, inflation, and output growth; \( A_0 \) being the impact matrix of the structural shocks at zero; \( B_1, \ldots, B_p \) being the AR matrices of the VAR; and \( \epsilon_t = A_0^{-1}u_t \)—where \( u_t \) is the \( N \times 1 \) vector containing the VAR’s reduced-form shocks—being a vector collecting the VAR’s structural innovations. The vector \( \epsilon_t \) is defined as \( \epsilon_t = [\epsilon_{R,t}, \epsilon_{-R,t}]' \), where \( \epsilon_{R,t} \) is the monetary policy shock, and \( \epsilon_{-R,t} \) is a vector collecting all the structural shocks other than \( \epsilon_{R,t} \). Let’s define \( \tilde{B}_0 \equiv A_0^{-1}, \ \tilde{B}_1 \equiv A_0^{-1}B_1, \ldots, \tilde{B}_p \equiv A_0^{-1}B_p \), and let’s partition \( \tilde{B}_0, \tilde{B}_1, \ldots, \tilde{B}_p \) as

\[ \tilde{B}_0 = \begin{bmatrix} \tilde{B}_0^R & \tilde{B}_0^X \\ \frac{1_{1 \times (N-1)}}{B_0^X} & \tilde{B}_0^Y \end{bmatrix}, \ \tilde{B}_1 = \begin{bmatrix} \tilde{B}_1^R & \tilde{B}_1^X \\ \frac{1_{1 \times N}}{B_1^X} & \tilde{B}_1^Y \end{bmatrix}, \ \ldots, \ \tilde{B}_p = \begin{bmatrix} \tilde{B}_p^R & \tilde{B}_p^X \\ \frac{1_{1 \times N}}{B_p^X} & \tilde{B}_p^Y \end{bmatrix} \]  

Leaving the short-term rate unchanged after the impact is then achieved by ‘zeroing out’ the relevant elements of the matrices \( \tilde{B}_0, \tilde{B}_1, \ldots, \tilde{B}_p \) in (14) as follows:

\[ \tilde{B}_{0}^{*} = \begin{bmatrix} \tilde{B}_{0,11}^{R} & 0_{1 \times (N-1)} \\ \frac{1_{1 \times (N-1)}}{B_0^X} & \tilde{B}_0^Y \end{bmatrix}, \ \tilde{B}_1^{*} = \begin{bmatrix} 0_{1 \times N} \\ \frac{1_{1 \times N}}{B_1^X} \end{bmatrix}, \ \ldots, \ \tilde{B}_p^{*} = \begin{bmatrix} 0_{1 \times N} \\ \frac{1_{1 \times N}}{B_p^X} \end{bmatrix} \]  

where \( \tilde{B}_{0,11}^{R} \) is the (1,1) element of \( \tilde{B}_0 \). The dynamics of the system after the initial impact is then described by the reduced-form VAR implied by \( \tilde{B}_0^{*}, \tilde{B}_1^{*}, \ldots, \tilde{B}_p^{*} \). Finally, starting from the ninth quarter after the impact, we allow the SVAR’s monetary rule to ‘kick in’, and we therefore use the original matrices \( B_1, \ldots, B_p \), rather than those implied by \( \tilde{B}_0^{*}, \tilde{B}_1^{*}, \ldots, \tilde{B}_p^{*} \).

Several findings are readily apparent from Figures 6-13. In particular,

- with the exception of the United States, the IRFs of the spread itself to a negative one-percent shock exhibit little time variation.

- On the contrary, evidence of time variation is, in general, quite substantial for both inflation and real GDP growth, thus providing both a strong justification for the use of time-varying methods, and an important caveat to results produced by fixed-coefficient models. This is especially apparent for the responses of U.S. inflation and GDP growth, which since the end of the 1960s have exhibited three peaks around the time of the Great Inflation of the 1970s, of the
recession of the early 1990s, and of the most recent period. These results clearly suggest that a fixed-coefficient model estimated over (say) the last two decades will understate the impact on inflation and output growth of a compression in the yield spread during the financial crisis, as this sample period mixes two sub-samples which, in this respect, are quite different. Evidence of time variation is even more apparent for the United Kingdom, for both inflation and output growth. In particular, for both variables the impact of a compression in the yield spread appears to have increased in recent years.

- Finally—and crucially, for the present purposes—the stimulative power on both inflation and output growth of a compression in the spread appears to be substantial. For the United States in 2009Q4, for example, annual real GDP growth increased (based on median estimates) by 1.3 percent in the quarter of impact, it peaks at 1.9 percent three quarters after the impact, and it then rapidly fades away over subsequent quarters. The impact on annual inflation starts at 0.3 percent on impact, it peaks at 1.1 percent after three quarters, and it then decreases. Results for the other countries are quantitatively slightly different but exhibit, overall, the same order of magnitude.

A caveat to these results  Although in principle entirely correct from the point of view of SVAR methodology, this way of computing the IRFs of interest suffers from the following shortcoming. Strictly speaking, the exercise we have just described is akin to a SVAR-based policy counterfactual, as it is based on the notion of taking an estimated SVAR and changing (some of) the parameters in its structural monetary policy rule (within the present context, setting them to zero). As such, as originally pointed out by Sargent (1979), it is vulnerable to the Lucas critique, and should be regarded, in general, as unreliable. Indeed, as shown by Benati and Surico (2009) by means of a single example based on an estimated standard New Keynesian model, and as extensively analysed by Benati (2010a) based on a battery of estimated DSGE models, the results produced by such counterfactuals may turn out to be misleading. In the next paragraph, we therefore perform the exercise under consideration by manipulating monetary policy shocks, rather than the coefficients of the SVAR’s monetary policy rule.

3.2.2 Results obtained based on the ‘constant-interest-rate’ projection methodology

We compute IRFs to a spread shock by choosing, for eight quarters after the impact, a sequence of monetary policy shocks such as to keep the short-term rate constant, thus exactly neutralising the impact of the systematic component of monetary policy, which would call (e.g.) for short-term rate increases in response to increases in inflation and output growth. This method is routinely used within central banks in order to compute ‘constant interest rate’ (henceforth, CIR) projections. Starting from the
ninth quarter after the impact, on the other hand, we allow the short rate to move according to what is dictated by the SVAR’s monetary rule. Two things ought to be stressed here.

First, this way of performing the exercise possesses one important element of understatement of the macroeconomic impact of a compression in the yield spread: from a DSGE model’s perspective, since the structural monetary rule which is encoded in the estimated SVAR predicts that the short-term rate always reacts to the state of the economy, this exercise ignores, by construction, the impact on agents’ expectations of the central bank’s announcement that it will keep the interest rate unchanged for an ‘extended period’.\footnote{We say ‘for an extended period’ since, as we previously pointed out, leaving the interest rate unchanged forever, and announcing that to the public, leads to global indeterminacy.} Therefore, our results most likely provide a lower bound for the macroeconomic effect of a compression in the yield spread.

Second, although, from a strictly technical point of view, this exercise is not vulnerable to the Lucas critique (different from the exercise of subsection 3.2.1, indeed, we are here uniquely manipulating shocks, rather than the coefficients of the structural VAR’s monetary rule), from a practical, substantive point of view things are unfortunately less clear-cut.\footnote{We wish to thank a referee for pointing this out.} The key point is that the interest rate is here consistently deviating in one direction—downwards—from the path that would be implied uniquely by the systematic component of monetary policy, due to a sequence of interest rate ‘surprises’ (i.e., shocks) all of the same sign. In order to be willing to assume that, under such circumstances, the public will not revise the model it uses to forecast the future path of the nominal interest rate, we must be ready to believe that it will remain oblivious to such a strong—in fact, perfect—pattern of autocorrelation of the policy shocks. If, on the other hand, the public were to notice that policy shocks were no longer drawn from a zero-mean, symmetric distribution, and they were rather being drawn from a distribution with an upper bound at zero, it would obviously use this information in order to generate its interest rate forecasts. So the key question becomes: ‘How reasonable is the assumption that the public will behave in such a myopic way under the present circumstances?’ Answering this question is not straightforward: in particular, the fact that the public will or will not detect such a pattern crucially depends on both, how large these shocks are, and how long they last. In the limit, the public will obviously detect large and prolonged deviations from the path which would be uniquely dictated by the systematic component of monetary policy. In the case of smaller deviations which last for a comparatively short period, on the other hand, the assumption that the public will not detect such a systematic pattern is less far-fetched.

Compared with the results obtained by ‘zeroing out’ the coefficients of the SVAR’s monetary rule, the results derived based on the CIR methodology suffer from a purely practical shortcoming: for some countries, and for some quarters, the IRFs we thus obtain exhibit a significant volatility, which, intuitively, is due to the workings of
the following ‘feedback loop’. On impact, a compression of the spread exerts an expansionary effect, thus raising both output growth and inflation. As a consequence, starting from the first quarter after the impact the SVAR’s monetary rule would call for an increase in the policy rate, in order to counter such expansionary effects. The negative monetary policy shock we choose in order to neutralise such a reaction of the short rate, thereby keeping it constant, exerts a further expansionary effect on inflation and output growth, thus compounding the initial impact of the spread shock.

So, depending on (i) how large the impact of a spread shock on inflation and output growth is, and (ii) how strongly monetary policy responds to inflation and output growth, this feedback loop may lead to highly volatile IRFs. On the other hand, in the limit case in which the short rate did not react to inflation and output growth, the feedback loop would simply not even ‘kick in’, and the problem would not exist. This implies that this problem is not a general one, but it rather may or may not be there depending on the specific structure of the economy at each point in time.

Figures 16-18 illustrate this, by plotting, for the U.S., the Euro area, and the U.K., the median IRFs computed based on this methodology (these figures are exactly comparable to Figures 8, 10, and 14). Results for the U.K. are quite remarkably similar to those produced based on the alternative methodology. Those for the Euro area exhibit a greater volatility for a few quarters, but other than that are, once again, in the same ‘ballpark’ as those reported in Figure 10. Results for the U.S., however, are, for several quarters, implausibly volatile, thus reflecting the workings of the previously discussed feedback loop.

3.2.3 On the sources of time variation

As previously discussed, based on either of the two methodologies we detect significant time variation in the economy’s response to a compression in the yield spread. Although identifying the sources of such time variation is clearly beyond the scope of this paper, one possible cause deserves to be at least briefly mentioned. Historically, changes in the yield spread for a given short rate have had a multiplicity of causes: shifts in long-term inflation expectations, changes in the liquidity premium, etc. Since it is at least possible to entertain the hypothesis that different underlying causes of changes in the yield spread may lead to a different pattern of responses of the economy—that is, to different impulse-response functions to a compression of the yield spread of a given magnitude—one obvious possibility for the identified changes over time in the pattern of IRFs is that such changes may simply result from a change in the ‘mixture’ of the underlying shocks leading to changes in the yield spread.

\[17\] We wish to thank a referee for pointing this out.
4 Conclusions

We have explored the macroeconomic impact of a compression in the long-term bond yield spread within the context of the Great Recession of 2007-2009 via a Bayesian time-varying parameter structural VAR. We have identified a ‘pure’ spread shock which, leaving the short-term rate unchanged by construction, has allowed us to characterise the macroeconomic consequences of a compression in the yield spread induced by central banks’ asset purchases within an environment in which the short rate cannot move because it is constrained by the zero lower bound. Two main findings stood out. First, in all the countries we have analysed (U.S., Euro area, Japan, and U.K.) a compression in the long-term yield spread exerts a powerful effect on both output growth and inflation. Second, conditional on available estimates of the impact of the FED’s and the Bank of England’s asset purchase programmes on long-term government bond yield spreads, our counterfactual simulations have indicated that both in the U.S. and in the U.K. unconventional monetary policy actions have been successful at averting significant risks both of deflation and of output collapses comparable to those that took place during the Great Depression.
References


A The Data

A.1 Euro area
Quarterly seasonally adjusted series for real GDP, the GDP deflator, and a short-term rate are from the European Central Bank’s database. The sample period is 1970:1-2008:4. The 5- and 10-year Euro area composite corporate yields for AAA-rated bonds are from *Reuters* (acronyms are C6645Y and C66410Y), and are both available for the period April 2002-March 2009. For the period before April 2002 we linked the two series to the monthly series for the 5- and 10-year government bond yields from *Reuters*. Over the period of overlapping (that is, after April 2002) the corporate and government bond yield series exhibit a remarkably close co-movement, with only a systematic difference of several basis points between the corporate yield series and the corresponding government yield one. So we rescaled the government bond series in such a way that its value in April 2002 be the same as the value taken by the corporate bond series, and we linked the two series. Given (i) our focus on the most recent quarters, and (ii) our use of a time-varying parameters VAR, the fact that before April 2002 we only have a reasonable proxy for the corporate yields, rather than the actual series of interest should not be regarded as problematic. We converted the monthly linked series to the quarterly frequency by taking averages within the quarter.

A.2 Japan
Quarterly seasonally adjusted series for real GDP, the GDP deflator, and the discount rate are from the *International Monetary Fund’s International Financial Statistics* database. The acronyms are 15899BVRZF..., 15899BIRZF..., and 15860...ZF..., and the sample period is 1957Q1-2008Q4.

A.3 United States
A monthly seasonally unadjusted series for the Federal Funds rate (acronym is FEDFUNDS) available for the period July 1954-March 2009, and a monthly series for the 10-year Treasury constant-maturity rate (acronym is GS10) available for April 1953-March 2009, are from the St. Louis FED’s database. We converted them to quarterly frequency by taking averages within the quarter. Quarterly seasonally adjusted series for real GDP the GDP deflator (acronyms are GDPC96 and GDPDEF), available for the period 1947Q1-2008Q4, are from the same database.

A.4 United Kingdom
Quarterly seasonally adjusted series for real GDP and the GDP deflator are from the *Office for National Statistics*, and are available since the first quarter of 1955. The
Treasury bill rate and the long-term government bond yield are from the International Monetary Fund’s International Financial Statistics database.

B Details of the Markov-Chain Monte Carlo Procedure

We estimate (1)-(10) via Bayesian methods. The next two subsections describe our choices for the priors, and the Markov-Chain Monte Carlo algorithm we use to simulate the posterior distribution of the hyperparameters and the states conditional on the data, while the third section lays out how we compute the generalised impulse-response functions.

B.1 Priors

For the sake of simplicity, the prior distributions for the initial values of the states—\( \theta_0, \alpha_0, h_0, \) and \( q_0 \)—which we postulate all to be normal, are assumed to be independent both from one another, and from the distribution of the hyperparameters. In order to calibrate the prior distributions for \( \theta_0, \alpha_0, h_0, \) and \( q_0 \) we estimate a time-invariant version of (1) based on the first 10 years of data, and we set

\[
\theta_0 \sim N(\hat{\theta}_{OLS}, 4 \cdot \hat{V}(\hat{\theta}_{OLS})) \tag{B1}
\]

As for \( \alpha_0 \) and \( h_0 \) we proceed as follows. Let \( \hat{\Sigma}_{OLS} \) be the estimated covariance matrix of \( \epsilon_t \) from the time-invariant VAR, and let \( C \) be the lower-triangular Choleski factor of \( \hat{\Sigma}_{OLS} \)—i.e., \( C' = \hat{\Sigma}_{OLS} \). We set

\[
\ln h_0 \sim N(\ln \mu_0, 10 \times I_4) \tag{B2}
\]

where \( \mu_0 \) is a vector collecting the squared elements on the diagonal of \( C \). We then divide each column of \( C \) by the corresponding element on the diagonal—and let’s call the matrix we thus obtain \( \tilde{C} \)—and we set

\[
\alpha_0 \sim N(\tilde{\alpha}_0, \tilde{V}(\tilde{\alpha}_0)) \tag{B3}
\]

where \( \tilde{\alpha}_0 \)—which, for future reference, we define as \( \tilde{\alpha}_0 = (\tilde{\alpha}_{0,11}, \tilde{\alpha}_{0,21}, \ldots, \tilde{\alpha}_{0,61})' \)—is a vector collecting all the non-zero and non-one elements of \( \tilde{C}^{-1} \) (i.e, the elements below the diagonal), and its covariance matrix, \( \tilde{V}(\tilde{\alpha}_0) \), is postulated to be diagonal, with each individual \( (j,j) \) element equal to 10 times the absolute value of the corresponding \( j \)-th element of \( \tilde{\alpha}_0 \). Such a choice for the covariance matrix of \( \alpha_0 \) is clearly arbitrary, but is motivated by our goal to scale the variance of each individual element of \( \alpha_0 \) in such a way as to take into account of the element’s magnitude.

As for \( q_0 \) we proceed as follows. Let \( Q_0 \) be the prior matrix for the extent of random-walk drift of the VAR’s parameters (that is, the random walks collected in
the vector \( \theta_t \) that we would use if we were working with a traditional Bayesian time-varying parameters VAR with a constant extent of random-walk drift over the sample. We set \( Q_0 = \gamma \times \Sigma_{OLS} \), with \( \gamma = 1.0 \times 10^{-4} \), the same value used in Primiceri (2005), and a relatively ‘conservative’ prior for the extent of drift compared (e.g.) to the \( 3.5 \times 10^{-4} \) used by Cogley and Sargent (2005). We set

\[
\ln \varphi_0 \sim N(10^{-2} \times \ln \varphi_0, 10 \times I_{N \times (1+N_p)}) \tag{B4}
\]

where \( \varphi_0 \) is a vector collecting the elements on the diagonal of \( Q_0 \).

Turning to the hyperparameters, we postulate independence between the parameters corresponding to the matrices \( S \) and \( Z \)—an assumption we adopt uniquely for reasons of convenience—and we make the following, standard assumptions. The three blocks of \( S \) are assumed to follow inverted Wishart distributions, with prior degrees of freedom set, again, equal to the minimum allowed, respectively, 2, 3 and 4:

\[
S_1 \sim IW(\tilde{S}^{-1}_1, 2) \tag{B5}
\]

\[
S_2 \sim IW(\tilde{S}^{-1}_2, 3) \tag{B6}
\]

\[
S_3 \sim IW(\tilde{S}^{-1}_3, 4) \tag{B7}
\]

As for \( \tilde{S}_1, \tilde{S}_2 \) and \( \tilde{S}_3 \), we calibrate them based on \( \tilde{\alpha}_0 \) in (B3) as \( \tilde{S}_1 = 10^{-3} \times |\tilde{\alpha}_{0,11}|, \tilde{S}_2 = 10^{-3} \times \text{diag}(|\tilde{\alpha}_{0,21}|, |\tilde{\alpha}_{0,31}|) \) and \( \tilde{S}_3 = 10^{-3} \times \text{diag}(|\tilde{\alpha}_{0,41}|, |\tilde{\alpha}_{0,51}|, |\tilde{\alpha}_{0,61}|) \). Such a calibration is consistent with the one we adopted for \( Q \), as it is equivalent to setting \( \tilde{S}_1, \tilde{S}_2 \) and \( \tilde{S}_3 \) equal to \( 10^{-4} \) times the relevant diagonal block of \( V(\tilde{\alpha}_0) \) in (B3). As for the variances of the innovations to the stochastic volatilities for the VAR’s reduced-form shocks, we follow Cogley and Sargent (2002, 2005) and we postulate an inverse-Gamma distribution for the elements of \( Z_\nu \),

\[
\sigma_{\nu,i}^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{1}{2}\right) \tag{B8}
\]

Finally, as for the variances of the innovations to the stochastic volatilities for the VAR’s random-walk parameters’ innovations, we postulate an inverse-Gamma distribution for the elements of \( Z_\omega \),

\[
\sigma_{\omega,i}^2 \sim IG\left(\frac{10^{-4}}{2}, \frac{10}{2}\right) \tag{B9}
\]

(B9) implies that the prior for \( \sigma_{\omega,i}^2 \) has the same mean as in Cogley, Primiceri, and Sargent (2010), but it has a smaller variance.

### B.2 Simulating the posterior distribution

We simulate the posterior distribution of the hyperparameters and the states conditional on the data via the following MCMC algorithm, combining elements of Primiceri (2005) and Cogley and Sargent (2002, 2005). In what follows, \( x_t \) denotes the
entire history of the vector $x$ up to time $t$—i.e. $x' = [x'_1, x'_2, \ldots, x'_t]$—while $T$ is the sample length.

(a) **Drawing the elements of $\theta_t$** Conditional on $Y^T$, $\alpha^T$, and $H^T$, the observation equation (1) is linear, with Gaussian innovations and a known covariance matrix. Following Carter and Kohn (2004), the density $p(\theta^T|Y^T, \alpha^T, H^T, V)$ can be factored as

$$p(\theta^T|Y^T, \alpha^T, H^T, V) = p(\theta_T|Y^T, \alpha^T, H^T, V) \prod_{t=1}^{T-1} p(\theta_t|\theta_{t+1}, Y^T, \alpha^T, H^T, V)$$  \hspace{1cm} (B10)

Conditional on $\alpha^T$, $H^T$, and $V$, the standard Kalman filter recursions nail down the first element on the right hand side of (B10), $p(\theta_T|Y^T, \alpha^T, H^T, V) = N(\theta_T, P_T)$, with $P_T$ being the precision matrix of $\theta_T$ produced by the Kalman filter. The remaining elements in the factorization can then be computed via the backward recursion algorithm found, e.g., in Kim and Nelson (2000), or Cogley and Sargent (2005, appendix B.2.1). Given the conditional normality of $\theta$, we have

$$\theta_{t|t+1} = \theta_{t|t} + P_{t|t}^{-1} (\theta_{t+1} - \theta_{t})$$  \hspace{1cm} (B11)

$$P_{t|t+1} = P_{t|t} - P_{t|t} P_{t+1|t} P_{t|t}$$  \hspace{1cm} (B12)

which provides, for each $t$ from $T$-1 to 1, the remaining elements in (1), $p(\theta_t|\theta_{t+1}, Y^T, \alpha^T, H^T, V) = N(\theta_{t|t+1}, P_{t|t+1})$. Specifically, the backward recursion starts with a draw from $N(\theta_T, P_T)$, call it $\tilde{\theta}_T$. Conditional on $\tilde{\theta}_T$, (B11)-(B12) give us $\theta_{T-1|T}$ and $P_{T-1|T}$, thus allowing us to draw $\tilde{\theta}_{T-1}$ from $N(\theta_{T-1|T}, P_{T-1|T})$, and so on until $t=1$.

(b) **Drawing the elements of $\alpha_t$** Conditional on $Y^T$, $\theta^T$, and $H^T$, following Primiceri (2005), we draw the elements of $\alpha_t$ as follows. Equation (1) can be rewritten as $A_t \tilde{Y}_t = A_t (Y_t - X_t' \theta_t) = A_t \epsilon_t = u_t$, with Var($u_t$) = $H_t$, namely

$$\tilde{Y}_{2,t} = -\alpha_{21,t} \tilde{Y}_{1,t} + u_{2,t}$$  \hspace{1cm} (B13)

$$\tilde{Y}_{3,t} = -\alpha_{31,t} \tilde{Y}_{1,t} - \alpha_{32,t} \tilde{Y}_{2,t} + u_{3,t}$$  \hspace{1cm} (B14)

$$\tilde{Y}_{4,t} = -\alpha_{41,t} \tilde{Y}_{1,t} - \alpha_{42,t} \tilde{Y}_{2,t} - \alpha_{43,t} \tilde{Y}_{3,t} + u_{4,t}$$  \hspace{1cm} (B15)

-plus the identity $\tilde{Y}_{1,t} = u_{1,t}$—where $[\tilde{Y}_{1,t}, \tilde{Y}_{2,t}, \tilde{Y}_{3,t}, \tilde{Y}_{4,t}]' \equiv \tilde{Y}_t$. Based on the observation equations (B13)-(B15), and the transition equation (8), the elements of $\alpha_t$ can then be drawn by applying the same algorithm we described in the previous paragraph separately to (B13)-(B15). The assumption that $S$ has the block-diagonal structure (10) is in this respect crucial, although, as stressed by Primiceri (2005, Appendix D), it could in principle be relaxed.

(c) **Drawing the elements of $H_t$** Conditional on $Y^T$, $\theta^T$, and $\alpha^T$, the orthogonalised innovations $u_t \equiv A_t (Y_t - X_t' \theta_t)$, with Var($u_t$) = $H_t$, are observable. Following
Cogley and Sargent (2002), we then sample the \( h_{i,t} \)'s by applying the univariate algorithm of Jacquier, Polson, and Rossi (1994) element by element.\(^{18}\)

\[ (d) \text{ Drawing the elements of } Q_t \quad \text{Conditional on } \theta^T, \text{ the innovations } \eta_t = \theta_t - \theta_{t-1}, \text{ with } \text{Var}(\eta_t) = Q_t, \text{ are observable, and, along the lines of point } (c), \text{ we therefore sample the } q_{j,t} \text{'s by applying the univariate algorithm of Jacquier, Polson, and Rossi (1994) element by element.} \]

\[ (e) \text{ Drawing the hyperparameters} \]

Finally, conditional on \( Y^T, \theta^T, H^T, \) and \( \alpha^T, \) the innovations to \( \theta_t, \alpha_t, \) the \( h_{i,t} \)'s and the \( q_{j,t} \)'s are observable, which allows us to draw the hyperparameters—the elements of \( S_1, S_2, S_3 \) and the \( \sigma_{\omega,i}^2 \) and the \( \sigma_{\omega,j}^2 \)—from their respective distributions.

Summing up, the MCMC algorithm simulates the posterior distribution of the states and the hyperparameters, conditional on the data, by iterating on \((a)-(e)\). In what follows, we use a burn-in period of 50,000 iterations to converge to the ergodic distribution, and after that we run 10,000 more iterations sampling every 10th draw in order to reduce the autocorrelation across draws.\(^{19}\)

### B.3 Computing generalised impulse-response functions

Here we describe the Monte Carlo integration procedure we use in Section 3.2 to compute generalised IRFs to a spread shock.

Randomly draw the current state of the economy at time \( t \) from the Gibbs sampler’s output. Given the current state of the economy, repeat the following procedure 100 times. Draw four independent \( N(0, 1) \) variates—the four structural shocks—and based on the relationship \( \epsilon_t = A_{0,t} \epsilon_t \), with \( \epsilon_t \equiv [\epsilon^M_t, \epsilon^{SP}_t, \epsilon^D_t, \epsilon^S_t] \), where \( \epsilon^M_t, \epsilon^{SP}_t, \epsilon^D_t, \) and \( \epsilon^S_t \) are the monetary policy, spread, demand non-policy, and supply structural shocks, respectively, compute the reduced-form shocks \( \epsilon_t \) at time \( t \). Simulate both the VAR’s time-varying parameters and the covariance matrix of its reduced-form innovations, \( \Omega_t \), 20 quarters into the future. Based on the simulated \( \Omega_t \), randomly draw reduced-form shocks from \( t+1 \) to \( t+20 \). Based on the simulated \( \theta_t \), and on the sequence of reduced-form shocks from \( t \) to \( t+20 \), compute simulated paths for the four endogenous variables. Call these simulated paths as \( \tilde{X}_{t,t+20} \), \( j = 1, \ldots, 100 \). Repeat the same procedure 100 times based on exactly the same simulated paths for the VAR’s time-varying parameters, the \( \theta_t \); the same reduced-form shocks at times \( t+1 \) to \( t+20 \); and the same structural shocks \( \epsilon^M_t, \epsilon^D_t, \) and \( \epsilon^S_t \) at time \( t \), but setting \( \epsilon^{SP}_t \) to one. Call these simulated paths as \( \tilde{X}_{t,t+20} \). For each of the 100 iterations define \( \text{ir}_j \tilde{X}_{t,t+20} \equiv \tilde{X}_{t,t+20} - \tilde{X}_{t,t+20} \). Finally, compute each of the 1,000 generalised IRFs as the mean of the distribution of the \( \text{ir}_j \tilde{X}_{t,t+20} \)'s.

\(^{18}\)For details, see Cogley and Sargent (2005, Appendix B.2.5).

\(^{19}\)In this we follow Cogley and Sargent (2005). As stressed by Cogley and Sargent (2005), however, this has the drawback of ‘increasing the variance of ensemble averages from the simulation’.
Figure 1  Checking for the convergence of the Markov chain: inefficiency factors for the draws from the ergodic distribution for the hyperparameters and the states (VAR for the United States with the 10-year Treasury bond yield spread)
Figure 2 Checking for the convergence of the Markov chain: inefficiency factors for the draws from the ergodic distribution for the hyperparameters and the states (VAR for the Euro area with the 10-year synthetic government bond yield spread)
Figure 3  Checking for the convergence of the Markov chain: inefficiency factors for the draws from the ergodic distribution for the hyperparameters and the states (VAR for Japan with the 10-year government bond yield spread)
Figure 4 Checking for the convergence of the Markov chain: inefficiency factors for the draws from the ergodic distribution for the hyperparameters and the states (VAR for the United Kingdom with the long-term bond yield spread)
Figure 5  GDP deflator inflation rates and estimated trends (medians and 16th and 84th percentiles)
Figure 6  Actual U.S. inflation and output growth during the Great Recession, and counterfactual inflation and output growth eliminating the impact on the spread of the FED’s asset purchases programs (conditional on the average estimate of the impact of asset purchases by Gagnon et al., 2010)
Figure 7  Actual U.K. inflation and output growth during the Great Recession, and counterfactual inflation and output growth assuming that the impact on the spread of the Bank of England’s asset purchases programs has been equal to 50 basis points
Figure 8 United States: median IRFs to a 1% negative shock to the 10-year Treasury bond yield spread, 1965Q4-2009Q4 (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 9  United States: IRFs to a 1% negative shock to the 10-year Treasury bond yield spread, selected quarters (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 10  Euro area: median IRFs to a 1% negative shock to the 10-year government bond yield spread, 1981Q1-2009Q3 (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 11  Euro area: median IRFs to a 1% negative shock to the 10-year government bond yield spread, selected quarters (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 12  Japan: median IRFs to a 1% negative shock to the 10-year corporate bond yield spread, 1976Q3-2009Q4 (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 13 Japan: median IRFs to a 1% negative shock to the 10-year corporate bond yield spread, selected quarters (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 14 United Kingdom: median IRFs to a 1% negative shock to the long-term bond yield spread, 1965Q4-2009Q4 (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 15  United Kingdom: median IRFs to a 1% negative shock to the long-term bond yield spread, selected quarters (computed setting to zero for 8 quarters the coefficients in the SVAR’s monetary rule)
Figure 16  United States: median IRFs to a 1% negative shock to the 10-year Treasury bond yield spread, 1965Q4-2009Q4 (based on the ‘constant-interest-rate’ projections methodology, keeping the short-term rate constant for 8 quarters after the impact)
Figure 17  Euro area: median IRFs to a 1% negative shock to the 10-year government bond yield spread, 1981Q1-2009Q3 (based on the ‘constant-interest-rate’ projections methodology, keeping the short-term rate constant for 8 quarters after the impact)
Figure 18 United Kingdom: median IRFs to a 1% negative shock to the long-term bond yield spread, 1965Q4-2009Q4 (based on the ‘constant-interest-rate’ projections methodology, keeping the short-term rate constant for 8 quarters after the impact)